School of Basic Sciences

Department of Basic Sciences
Mid Term Examination

Exam Date: 29 Sep 2023
Time : 90 Minutes
Marks : 50

Sem III - MSCM303 - Integral Equations and Calculus of Variation

Your answer should be specific to the question asked
Draw neat labeled diagrams wherever necessary

1) Explain the solution of the integral equation $x^3 = \int_0^x (x-t)^2 y(t)dt$ — K2 (2)

2) Find the solution of the integral equation $y(x) + \lambda \int_0^1 \sin xt \; y(t)dt = 1$ — K1 (3)

3) Estimate the integral equation corresponding to the following differential equation with initial conditions: — K2 (4)

$y''' - 2xy = 0;$
$y(0) = 1/2;$
$y' = y''(0) = 1$

4) Show that $R(x,t;\lambda) = e^{\lambda(x-t)}$ is resolvent kernel of the Volterra integral equation with the kernel $K(x,t) = 1$ — K2 (6)

5) Develop the resolvent kernel, $R(x,t;\lambda)$ of the following kernels for specified a and b $K(x,t) = x^2t^2;\; a = -1, b = 1$ — K3 (6)

6) Solve the given integral equation by applying Laplace transform: $Y(x) = t + 2\int_0^t Y(x)\cos(t-x)dx$ — K3 (9)

7) Examine the solution of the Abel's equation: $x^2 + x = \int_0^x \frac{y(t)dt}{(x^2-t)^{1/3}}$ — K4 (8)

8) Analyze the Eigen values and Eigen functions of the homogeneous equation $y(x) = \lambda \int_0^\pi K(x,t)y(t)dt$ $where K(x,t) = \begin{cases} \cos x \; sint, & 0 \le x \le t \\ \cos t \sin x, & t \le x \le \pi \end{cases}$ — K4 (12)

OR

Analyze the solution of the integral equation: $y(x) = 1 + \frac{1}{\pi}\int_0^{2\pi} \sin(x+t)y(t)dt$, which possesses infinite many solutions. — K4 (12)