OUTP-95-16P (rev)

REVIEW ARTICLE

Big Bang nucleosynthesis and physics beyond the Standard Model

Subir Sarkar

Department of Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, U.K.

Abstract. The Hubble expansion of galaxies, the 2.73 K blackbody radiation background and the cosmic abundances of the light elements argue for a hot, dense origin of the universe — the standard Big Bang cosmology — and enable its evolution to be traced back fairly reliably to the nucleosynthesis era when the temperature was of $\mathrm{O}(1)$ MeV corresponding to an expansion age of $\mathrm{O}(1)$ sec. All particles, known and hypothetical, would have been created at higher temperatures in the early universe and analyses of their possible effects on the abundances of the synthesized elements enable many interesting constraints to be obtained on particle properties. These arguments have usefully complemented laboratory experiments in guiding attempts to extend physics beyond the Standard $SU(3)_{\mathrm{c}}\otimes SU(2)_{\mathrm{L}}\otimes U(1)_Y$ Model, incorporating ideas such as supersymmetry, compositeness and unification. We first present a pedagogical account of relativistic cosmology and primordial nucleosynthesis, discussing both theoretical and observational aspects, and then proceed to examine such constraints in detail, in particular those pertaining to new massless particles and massive unstable particles. Finally, in a section aimed at particle physicists, we illustrate applications of such constraints to models of new physics.

PACS numbers: 98.80.Cq 14.60.-z 14.80.-j 95.30.-Cq

Short title: BBN and physics beyond the SM

Submitted to: Institute of Physics Publishing

February 1, 2008

† *Dedicated to Dennis Sciama on his 67th birthday*

Contents

1. Introduction

There has been interest in problems at the interface of cosmology and particle physics for over thirty years (see Zel'dovich 1965), but it is only in the past decade or so that the subject has received serious attention (see Börner 1988, Collins *et al* 1989, Kolb and Turner 1990, Ellis 1993). Cosmology, once considered to be outside the mainstream of physics and chiefly of interest to astronomers and applied mathematicians, has become a *physical* subject, largely due to the advances which have been made on the observational front (see Weinberg 1972, Peebles 1993). It has become increasingly clear that particle physicists can no longer afford to ignore the cosmological "laboratory", which offers a powerful probe of new physical phenomena far beyond the reach of terrestrial laboratories (see Steigman 1979, Dolgov and Zel'dovich 1981). Cosmological phenomena have thus come under detailed scrutiny by particle physicists, prompting deeper theoretical analyses (see Weinberg 1980, Wilczek 1991) as well as ambitious observational programmes (see Sadoulet 1992, Kolb and Peccei 1995).

The increasing interaction between particle physics and cosmology has largely resulted from the establishment of 'standard models' in both fields which satisfactorily describe all known phenomena but whose very success, paradoxically, establishes them as intrinsically incomplete pictures of physical reality. Our reconstruction of the history of the universe in figure 1 emphasizes the interdependence of these models. The familiar physics of electromagnetism, weak interactions and nuclear reactions provide a sound basis for the standard Big Bang cosmology up to the beginning of the nucleosynthesis era, when the universe was about 10^{-2} sec old. The Standard $SU(3)_{\rm c}\otimes SU(2)_{\rm L}\otimes U(1)_Y$ Model (SM) of particle physics (see Cheng and Li 1984, Kane 1987), brilliantly confirmed by all experiments to date (see Burkhardt and Steinberger 1991, Bethke and Pilcher 1992), allows us to extrapolate back further, to $t \sim 10^{-12}$ sec. Two phase transitions are believed to have occurred in this interval, although a detailed understanding of their dynamics is still lacking (see Kapusta 1988, Yaffe 1995). The first is associated with the confinement of quarks into hadrons and chiral symmetry breaking by the strong interactions at $T_{\rm c}^{\rm qh} \sim \Lambda_{\rm QCD} \approx 200\,{\rm MeV}$, and the second with the spontaneous breaking of the unified electroweak symmetry to electromagnetism, $SU(2)_{\rm L}\otimes U(1)_Y \to U(1)_{\rm em}$ at $T_{\rm c}^{\rm EW} \sim 250\,{\rm GeV}$, when all known particles received their masses through the Higgs mechanism. To go beyond this point requires an extension of the SM; indeed, the very success of the model demands such new physics.

Similarly, the standard cosmological model of an adiabatically expanding, homogeneous and isotropic universe requires extreme fine tuning of the initial conditions of the Big Bang, as emphasized by Dicke and Peebles (1979). The problem essentially consists of explaining why the universe is as old ($\gtrsim 3\times10^{17}$ sec) or as large ($\gtrsim 10^{28}$ cm) as it is today, relative to the Planck time (5.39×10^{-44} sec) or the Planck length

(1.62×10^{-33} cm), which are the appropriate physical scales governing gravitational dynamics. Although a resolution of this may have to await progress in our understanding of quantum gravity (see Penrose 1979, 1989), there has been enthusiastic response to the simpler solution proposed by Guth (1981), viz. that there was a period of non-adiabatic accelerated expansion or 'inflation', possibly associated with a phase transition in the early universe (see Linde 1990). This has the additional advantage that it naturally generates a nearly scale-invariant 'Harrison-Zel'dovich' spectrum of scalar density fluctuations (see Mukhanov *et al* 1992) which can seed the growth of the observed large-scale structure in the expanding universe (see Efstathiou 1990, Padmanabhan 1993). Another fundamental problem of the standard cosmology is that the observed abundance of baryonic matter is $\sim 10^9$ times greater than the relic abundance expected from a state of thermal equilibrium, while no antimatter is observed (see Steigman 1976), thus requiring a primordial asymmetry between matter and anti-matter. To generate this dynamically requires new physics to violate baryon number (B) and charge-parity (CP) at high temperatures, in an out-of-equilibrium situation to ensure time asymmetry (Sakharov 1967, see Cohen *et al* 1994, Rubakov and Shaposhnikov 1996). More recently, it has been recognized that baryons are probably a minor constituent of the universe, since all observed structures appear to be dominated by dark matter (see Binney and Tremaine 1987, Peebles 1993) which is probably non-baryonic. The growing interest in the early universe stems from the realization that extensions of physics beyond the SM naturally provide the mechanisms for processes such as inflation and baryogenesis, as well as new particle candidates for the dark matter. These exciting developments have been discussed in a number of schools and conferences (see e.g. Gibbons *et al* 1983, Setti and Van Hove 1984, Kolb *et al* 1986b, Piran and Weinberg 1986, Alvarez *et al* 1987, Hinchliffe 1987, De Rújula *et al* 1987, Unruh and Semenoff 1988, Yoshimura *et al* 1988, Peacock *et al* 1990, Nilsson *et al* 1991, Sato and Audoze 1991, Nanopoulos 1991, Sanchez and Zichichi 1992, Akerlof and Srednicki 1993, Astbury *et al* 1995).

Such new physics is in fact necessary to address the theoretical shortcomings of the Standard Model itself (see Ross 1984, Mohapatra 1992). Its phenomenological success requires that the Higgs boson, which gives masses to all known particles, cannot itself be much more massive than its vacuum expectation value (VEV) which sets the electroweak ('Fermi') scale, $v \equiv (\sqrt{2}G_{\rm F})^{-1/2} = 246\,{\rm GeV}$. This creates the 'naturalness' or 'hierarchy' problem, viz. why is the Higgs mass not pushed up to the Planck mass ($M_{\rm P} \equiv G_{\rm N}^{-1/2} = 1.221 \times 10^{19}\,{\rm GeV}$) due to the *quadratically* divergent radiative corrections it receives due to its couplings to all massive particles?† Supersymmetry (SUSY) addresses this problem by imposing a symmetry between bosons and fermions

† By contrast, it is 'natural' for fermions to be light relative to the Planck scale since letting their masses go to zero reveals a chiral symmetry which tames the radiative corrections to be only logarithmically divergent; there is no such symmetry to 'protect' the mass of a scalar Higgs boson.

which makes such radiative corrections cancel to zero. This requires all known particles (boson/fermion) to have supersymmetric (fermion/boson) partners distinguished by a new quantum number called R-parity; if it is conserved, the lightest supersymmetric particle would be stable. Supersymmetry must be broken in nature since known particles do not have supersymmetric partners of the same mass. However the Higgs mass would still be acceptable if the scale of SUSY breaking (hence the masses of the supersymmetric partners) is not much beyond the Fermi scale. When such breaking is realized *locally*, as in gauge theories, a link with general coordinate transformations, i.e. gravity, emerges; this is supergravity (SUGRA) (see Van Nieuwenhuizen 1981, Wess and Bagger 1993). Technicolour is an alternative approach in which the offending *elementary* Higgs particle is absent (see Farhi and Susskind 1981, Kaul 1983); electroweak symmetry breaking is now seen as a dynamic phenomenon (see King 1995, Chivukula *et al* 1995), akin to the breaking of chiral symmetry by the strong interactions. However no realistic technicolour model has been constructed satisfying all experimental constraints, in particular the small radiative corrections to SM parameters measured at *LEP* (see Lane 1993).

Another conundrum is that CP is known to be well conserved by the strong interactions, given the stringent experimental upper limit on the neutron electric dipole moment, whereas QCD, the successful theory of this interaction, contains an arbitrary CP violating parameter. An attractive solution is to replace this parameter by a field which dynamically relaxes to zero — the axion (see Kim 1987, Cheng 1988). This is a pseudo-Goldstone boson generated by the breaking of a new global $U(1)$ 'Peccei-Quinn' symmetry at a scale f_a (see Peccei 1989). This symmetry is also explicitly broken by QCD instanton effects, hence the axion acquires a small mass $m_a \sim f_\pi^2/f_a$ when the temperature drops to $T \sim \Lambda_{\rm QCD}$. The mixing with the pion makes the axion unstable against decay into photons; negative experimental searches for decaying axions then constrain f_a to be beyond the Fermi scale, implying that axions are light enough to be produced in stellar interiors. Considerations of stellar cooling through axion emission imply $f_a \gtrsim 10^{10}$ GeV, which *requires* the axion (if it exists!) to have an interesting cosmological relic density (see Raffelt 1990, Turner 1990).

Yet another motivation for going beyond the Standard Model is the unification of forces. Grand unified theories (GUTs) of the strong and electroweak interactions at high energies also provide a physical need for inflation in order to dilute the embarrassingly large abundance of magnetic monopoles expected to be created during the breaking of the unified symmetry (see Preskill 1984). Unification naturally provides for baryon and lepton number violation (see Langacker 1981, Costa and Zwirner 1986) which allows for generation of the cosmological baryon asymmetry (see Kolb and Turner 1983) as well as masses for neutrinos (see Mohapatra and Pal 1991). Recent data from *LEP* on the evolution of the gauge interaction couplings with energy indicate that such unification can occur at $M_{\rm GUT} \approx 2 \times 10^{16}$ GeV, but only in a (broken) supersymmetric theory with superparticle masses at around the Fermi scale (see Dimopoulos 1994, Ellis 1995). Moreover in such unified models, electroweak symmetry breaking via the Higgs mechanism is driven quite naturally by supersymmetry breaking (see Ibáñez and Ross 1993). A dynamical understanding of how supersymmetry itself is broken is expected to come from the theory of superstrings, the most ambitious attempt yet towards a finite quantum theory of gravity and its unification with all other forces (see Green *et al* 1987). Following the initial euphoria over the discovery of the anomaly-free heterotic superstring, progress has been difficult due to the problems of relating low energy physics to the higher dimensional world which is the natural domain of the string. However explicit examples of compactified four-dimensional strings have been constructed which reduce to a supersymmetric version of the Standard Model at low energies and also contain additional gauge bosons and gauge singlets which have only gravitational couplings to matter (see Dine 1988, 1990, Ibáñez 1994). Furthermore, the recent exciting discoveries of the special 'duality' properties of string theories (see Giveon *et al* 1994, Polchinski 1996) have begun to provide important insights into the issue of supersymmetry breaking (see Zwirner 1996).

It is thus a common feature of new physics beyond the Fermi scale to predict the existence of new particles which are unstable in general but some of which may be stable by virtue of carrying new conserved quantum numbers. Moreover their generic weak interactions ensure a cosmologically significant relic density (see Primack *et al* 1988, Turner 1991). In addition, known particles such as neutrinos, although strictly massless in the Standard Model, may acquire masses from such new physics, enabling them also to be candidates for dark matter. Conventionally, particle physicists look for new physics either by directly attempting to produce the new particles in high energy collisions at accelerators or by looking for exotic phenomena such as nucleon instability or neutrino masses. In this context, the standard cosmology, in particular Big Bang nucleosynthesis (BBN), provides an important new testing ground for new physics and, indeed, in many cases, provides the *only* "experimental" means by which the properties of new particles may be restricted (see Sarkar 1985, 1986). Whether or not one finds this satisfactory from a philosophical point of view, it is essential for this enterprise that we have the best possible understanding of the cosmological laboratory. This review presents the current status and is primarily aimed at particle physicists, although it is hoped that astrophysicists and cosmologists will also find it useful.

A decade or more ago, it was possible for reviewers (e.g. Steigman 1979, Dolgov and Zel'dovich 1981) to give a comprehensive discussion of all constraints on fundamental physics from cosmological considerations and many of the key papers could be found in one collection (Zee 1982). Subsequently several hundred papers on this subject have been published. For reasons of space we will restrict ourselves to a discussion of the constraints which follow from BBN alone but, for completeness, present all the neccessary cosmological background — both theory and observations. Rather than engage in a detailed critique of every published work, we present a pedagogical discussion of the basic physics, together with a summary of the key observational inputs, so that readers can assess the reliability of these constraints. Raffelt (1990) has presented a model review of this form which deals with astrophysical methods for constraining novel particle phenomena. A similar discussion of all types of cosmological constraints, including those deduced from the observed Hubble expansion and the 2.73 K blackbody radiation as well as other radiation backgrounds, will appear in Sarkar (1997).

We begin by outlining in section 2 the basic features of the standard Big Bang cosmological model and then discuss the thermodynamics of the early radiation-dominated era. In section 3 we present the essential physics of the BBN era and then discuss the observational data in some detail, highlighting the sources of uncertainty. We argue for the consistency of the standard model and briefly mention possible variations. This sets the stage for deriving general constraints in section 4 on both relativistic and non-relativistic hypothetical particles which may be present during nucleosynthesis. Finally, for the benefit of particle physicists we illustrate in section 5 how such cosmological arguments have complemented experimental searches for physics beyond the Standard Model, particularly in the neutrino sector, and also provided entirely new probes of such physics, e.g. technicolour and supersymmetry. We also discuss the implications for the nature of the dark matter.

It appears to be a widely held belief that cosmological data are not particularly accurate and the associated errors uncertain, so that the derived constraints cannot compare in reliability with those obtained in the laboratory. Although not entirely incorrect, this view is being increasingly challenged by modern observations; for example measurements of the background radiation temperature and anisotropy, the cosmic abundance of helium *et cetera* are now routinely quoted to several significant figures. Correspondingly there has been a growing appreciation of the systematic effects involved in the analysis of cosmological observations and careful attempts at their estimation. More importantly, cosmological data, even if more imprecise than accelerator data, are often much more *sensitive* to novel particle phenomena; for example, even a crude upper limit on the present energy density of the universe suffices to bound the masses of relic neutrinos to a level which improves by several orders of magnitude over precise laboratory experiments. Nevertheless, one should be cautious about rejecting an interesting theoretical possibility on the basis of a restrictive cosmological constraint (e.g. the bound on the number of neutrino-like particles present during BBN) without a critical appreciation of the underlying assumptions. We have tried wherever possible to clarify what these assumptions are and to refer to expert debate on the issues involved. (In writing down numerical values where errors are not quoted, the symbols $\sim$, $\approx$ and $\simeq$ indicate equality to within a factor of 10, factor of 2 and 10%, respectively.)

Due to space limitations, the references are not comprehensive but do include the seminal papers and recent reviews from which the intervening literature can be traced. We have used 'natural' units ($\hbar = c = k_{\rm B} = 1$) although astronomical units such as year, megaparsec or Solar mass are given where convenient. (For reference, $1\,{\rm GeV}^{-1} = 1.973 \times 10^{-14}\,{\rm cm} = 6.582 \times 10^{-25}\,{\rm sec}$, $1\ {\rm GeV} = 1.160 \times 10^{13}\,{\rm K} = 1.783 \times 10^{-24}\,{\rm gm}$, $1\ {\rm Mpc} = 3.086 \times 10^{24}\,{\rm cm}$, $1\ {\rm yr} = 3.156 \times 10^{7}\,{\rm sec}$, $1 M_\odot = 1.989 \times 10^{33}\,{\rm gm}$.) Astronomical quantities are listed in Allen (1973) and Lang (1992), while clarification of unfamiliar astrophysical terms may be sought in the excellent textbooks by Shu (1981), Mihalas and Binney (1981) and Longair (1981).

2. The standard cosmology

The standard Big Bang cosmological model assumes that the universe is spatially homogeneous and isotropic, an assumption originally dignified as the 'Cosmological Principle' (Milne 1935). Subsequently cosmological observations have provided empirical justification for this assumption as reviewed by Peebles (1980). Astronomical observations in the last decade have required a reappraisal of this issue with the discovery of cosmic structures on very large spatial scales. However careful studies of the clustering of galaxies and of the small angular fluctuations in the $2.73\,{\rm K}$ cosmic microwave background (CMB) have established (see Peebles 1993) that the universe is indeed homogeneous when averaged on scales exceeding a few hundred Mpc, out to spatial scales comparable to its present "size" (2.19) of several thousand Mpc.

2.1. The Friedmann-Lemaître-Robertson-Walker models

Homogeneity and isotropy considerably simplify the mathematical description of the cosmology since all hypersurfaces with constant cosmic standard time † are then maximally symmetric subspaces of the whole of space-time and all cosmic tensors (such as the metric $g_{\mu\nu}$ or energy-momentum $T_{\mu\nu}$) are form-invariant with respect to the isometries of these surfaces (see Weinberg 1972). These symmetries enable a relatively simple and elegant description of the dynamical evolution of the universe. Although the mathematical complexities of general relativity do allow of many exotic possibilities (see Hawking and Ellis 1973), these appear to be largely irrelevant to the physical universe, except perhaps at very early epochs. There are many pedagogical accounts of relativistic cosmology; to keep this review self-contained we reiterate the relevant points.

† Spatial coordinates may be defined through observables such as the apparent brightness or redshift, while time may be defined as a definite (decreasing) function of a cosmic scalar field such as the proper energy density ρ or the blackbody radiation temperature T, which are believed to be monotonically decreasing everywhere due to the expansion of the universe. Knowledge of the function $t = t(T)$ requires further assumptions, for example that the expansion is adiabatic.

For a homogeneous and isotropic evolving space-time, we can choose comoving spherical coordinates (i.e. constant for an observer expanding with the universe) in which the proper interval between two space-time events is given by the Robertson-Walker (R-W) metric

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \mathrm{d}t^2 - R^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2)\right]. \qquad (2.1)$$

Here $R(t)$ is the cosmic scale-factor which evolves in time describing the expansion (or contraction) of the universe and k is the 3-space curvature signature which is conventionally scaled (by tranforming $r \to |k|^{1/2}r$ and $R \to |k|^{-1/2}R$) to be -1, 0 or $+1$ corresponding to an elliptic, euclidean or hyperbolic space.†

The energy-momentum tensor is then required to be of the 'perfect fluid' form

$$T_{\mu\nu} = pg_{\mu\nu} + (p+\rho)u_\mu u_\nu \ , \qquad (2.2)$$

in terms of the pressure p, the energy density ρ and the four-velocity $u_\mu \equiv \mathrm{d}x_\mu/\mathrm{d}s$. (Here and below, we follow the sign conventions of Weinberg (1972).) The Einstein field equations relate $T_{\mu\nu}$ to the space-time curvature $R_{\lambda\mu\nu\kappa}$:

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R_c = -\frac{8\pi T_{\mu\nu}}{M_\mathrm{P}^2} \ , \qquad (2.3)$$

where $R_{\mu\nu}{\equiv}g^{\lambda\kappa}R_{\lambda\mu\kappa\nu}$ is the Ricci tensor and $R_c{\equiv}g^{\mu\nu}R_{\mu\nu}$ is the curvature scalar. For the present case these equations simplify to yield the Friedmann-Lemaître (F-L) equation for the normalized expansion rate H, called the Hubble parameter,

$$H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi\rho}{3M_\mathrm{P}^2} - \frac{k}{R^2} \ , \qquad (2.4)$$

as well as an equation for the acceleration

$$\ddot{R} = -\frac{4\pi\rho}{3M_\mathrm{P}^2}(\rho + 3p)R \ . \qquad (2.5)$$

Further, the conservation of energy-momentum

$$T^{\mu\nu}_{;\nu} = 0 \ , \qquad (2.6)$$

implies ‡

$$\frac{\mathrm{d}(\rho R^3)}{\mathrm{d}R} = -3pR^2. \qquad (2.7)$$

† This does not however fix the global topology; for example Euclidean space may be $\mathbf{R^3}$ and infinite or have the topology of a 3-torus ($\mathbf{T^3}$) and be finite in extent; however the latter possibility has recently been severely constrained by the non-observation of the expected characteristic pattern of fluctuations in the CMB (Starobinsky 1993, Stevens *et al* 1993).

‡ This does not imply conservation of the energy of matter since ρR^3 clearly decreases (for positive p) in an expanding universe due to work done against the gravitational field. In fact, we cannot in general even define a conserved total energy for matter plus the gravitational field unless space-time is asymptotically Minkowskian, which it is *not* for the R-W metric (see Witten 1981a).

This can also be derived from (2.4) and (2.5) since (2.3) and (2.6) are related by the Bianchi identities:

$$\left(R^{\mu\nu} - \tfrac{1}{2} g^{\mu\nu} R_c\right)_{;\mu} = 0 \ . \tag{2.8}$$

In principle we can add a cosmological constant, $\Lambda g_{\mu\nu}$, to the field equation (2.3), which would appear as an additive term $\Lambda/3$ on the RHS of the F-L equations (2.4) and (2.5). This is equivalent to the freedom granted by the conservation equation (2.6) to scale $T_{\mu\nu} \rightarrow T_{\mu\nu} + \Lambda g_{\mu\nu}$, so that Λ can be related to the energy-density of the vacuum (see Weinberg 1989):

$$\langle 0 \mid T_{\mu\nu} \mid 0 \rangle = -\rho_{\rm v} g_{\mu\nu} \ , \qquad \Lambda = \frac{8\pi\rho_{\rm v}}{M_{\rm P}^2} \ . \tag{2.9}$$

Empirically Λ is consistent with being zero today; in natural units $\Lambda < 10^{-120} M_{\rm P}^{-2}$ (see Carroll *et al* 1992). However the present vacuum is known to violate symmetries of the underlying gauge field theory, e.g. the $SU(2)_{\rm L} \otimes U(1)_Y$ symmetry of the electroweak interaction and (very probably) the symmetry unifying the $SU(3)_{\rm c}$ and electroweak interactions in a GUT (see Ross 1984). These symmetries would have been presumably restored at sufficiently high temperatures in the early universe and a finite value of Λ associated with the symmetric or false vacuum (see Linde 1979). (There are also other ways, not associated with symmetry breaking, in which the universe may have been trapped in a false vacuum state.) This possibility is exploited in the inflationary universe model of Guth (1981) and its successors (see Linde 1984, 1990, Olive 1990a), wherein the (approximately constant) vacuum energy drives a huge increase of the scale-factor during the transition to the true vacuum and is then converted during 'reheating' into interacting particles, thus accounting for the large entropy content of the universe, which is otherwise unexplained in the standard cosmology.

Knowing the equation of state, $p = p(\rho)$, ρ can now be specified as a function of R. For non-relativistic particles ('matter' or 'dust') with $p/\rho \approx T/m \ll 1$,

$$\rho_{\rm NR} \propto R^{-3}, \tag{2.10}$$

reflecting the dilution of density due to the increasing proper volume. For relativistic particles ('radiation') with $p = \rho/3$, an additional factor of R^{-1} enters due to the redshifting of the momentum by the expansion:

$$\rho_{\rm R} \propto R^{-4}. \tag{2.11}$$

In the modern context, it is also relevant to consider the contribution of 'vacuum energy' (i.e. a cosmological constant) for which the equation of state, dictated by Lorentz-invariance of the energy-momentum tensor, is $p = -\rho$, i.e.

$$\rho_{\rm v} \propto \text{constant} \, . \tag{2.12}$$

This completes the specification of the ensemble of Friedmann-Lemaître-Robertson-Walker (F-L-R-W) models. (As a historical note, Friedmann presented the dynamical equation (2.4) only for the case of pressureless dust, while Lemaître extended it to include the case of radiation and also wrote down the conservation equation (2.7).)

Taking $\Lambda = 0$, the curvature term k/R^2 in (2.4) is positive, zero or negative according as ρ is greater than, equal to or less than the critical density

$$\rho_{\rm c} = \frac{3H^2 M_{\rm P}^2}{8\pi} \equiv \frac{\rho}{\Omega} \ , \tag{2.13}$$

where Ω is the density parameter. The critical density today is †

$$\rho_{{\rm c}_0} = (2.999 \times 10^{-12} h^{1/2} \ {\rm GeV})^4 = 1.054 \times 10^{-5} h^2 \, {\rm GeV \, cm^{-3}} \ , \tag{2.14}$$

where h, the Hubble constant, is defined in terms of the present expansion rate,

$$h \equiv \frac{H_0}{100 \, {\rm km \, sec^{-1} \, Mpc^{-1}}} \ , \qquad H_0 \equiv \frac{\dot{R}_0}{R_0} \ . \tag{2.15}$$

The extragalactic distance scale is set by H_0 since a measured redshift

$$z \equiv \frac{\lambda(t_0) - \lambda(t)}{\lambda(t)} = \frac{R(t_0)}{R(t)} - 1 \tag{2.16}$$

is assumed to correspond to the distance $d \simeq z/H_0$. (This is an approximate relationship, since it is the recession velocity, not the redshift, which is truly proportional to distance for the R-W metric (see Harrison 1993), hence corrections are neccessary (see Weinberg 1972) for cosmologically large distances.) The major observational problem in obtaining H_0 is the uncertainty in determining cosmological distances (see Rowan-Robinson 1985, 1988, Jacoby *et al* 1992, Huchra 1992, Van den Bergh 1992, 1994, Fukugita *et al* 1993). Different estimates, while often inconsistent within the stated errors, generally fall in the range $40 - 100 \, {\rm km \, sec^{-1} Mpc^{-1}}$, i.e.

$$0.4 \lesssim h \lesssim 1 \ . \tag{2.17}$$

The *Hubble Space Telescope* has recently provided the means to directly calibrate various techniques through observations of Cepheid variables in distant galaxies (see Kennicutt *et al* 1995). According to the review by Hogan (1996), the central values obtained by the most reliably calibrated methods lie in the range $0.65 - 0.85$, although there is, as yet, no consensus among observers. This issue may soon be resolved by new techniques such as measurements of time delays between variations in multiple images of gravitationally lensed quasars (Kundic *et al* 1995, see Blandford and Narayan 1992) or of the 'Sunyaev-Zel'dovich' effect on the CMB by the X-ray emitting plasma in clusters of galaxies (Birkinshaw and Hughes 1994, see Rephaeli 1990), which bypass the traditional error-prone construction of the 'cosmological distance ladder'.

† The subscript $_0$ on any quantity denotes its present value.

Since $(\rho + 3p)$ is positive for both matter and radiation, $\ddot{R}$ is always negative (see (2.5)), hence the present age is bounded by the Hubble time

$$t_0 < H_0^{-1} = 9.778 \times 10^9 \; h^{-1} \, \mathrm{yr} \; , \tag{2.18}$$

corresponding to a present Hubble radius of

$$R_{\mathrm{H}}(t_0) = H^{-1}(t_0) \simeq 3000 \; h^{-1} \, \mathrm{Mpc} \; , \tag{2.19}$$

which sets the *local* spatial scale for the universe. Another scale, which depends on the past evolutionary history, is set by the finite propagation velocity of light signals. Consider a ray emitted at time t which has just reached us at time t_0:

$$\begin{aligned} \int_0^r \frac{\mathrm{d}r'}{\sqrt{1 - kr'^2}} &= \int_t^{t_0} \frac{\mathrm{d}t'}{R(t')} \\ &= \int_{R(t)}^{R(t_0)} \frac{\mathrm{d}R}{R} \left(\frac{8\pi\rho R^2}{3M_{\mathrm{P}}^2} - k \right)^{1/2} \; . \end{aligned} \tag{2.20}$$

Since $\rho R^2 \to \infty$ as $R \to 0$, for both non-relativistic and relativistic particles, the above integral converges as $t \to 0$. This indicates (see Rindler 1977) that there are sources from which light has not yet reached us, which are said to lie beyond our particle horizon, at proper distance

$$d_{\mathrm{H}}(t_0) = R(t_0) \int_0^{t_0} \frac{\mathrm{d}t'}{R(t')} = \kappa t_0 \; , \tag{2.21}$$

where $\kappa = 2, 3$ for $\rho = \rho_{\mathrm{R}}, \rho_{\mathrm{NR}}$ (taking $k = 0$). This creates a problem for the standard cosmology because looking back to earlier times we observe regions which were outside each other's (shrinking) horizons, but which nevertheless appear to be well-correlated. Consider the photons of the 2.73 K microwave background radiation which have been propagating freely since $z \approx 1000$; the particle horizon at that epoch subtends only $\approx 1^0$ on the sky, yet we observe the radiation arriving from all directions to have the same temperature to within 1 part in about 10^5. This problem too is solved in the inflationary universe (Guth 1981) where the energy density becomes dominated by a positive cosmological constant (2.12) at early times. The *accelerated* growth of $R(t)$ ($\ddot{R} > 0$ for $p = -\rho$) then rapidly blows up a region small enough to be causally connected at that time into the very large universe we see today.

Returning to the standard cosmology, the future evolution is determined by the sign of k, or equivalently, the value of Ω (assuming $\Lambda = 0$). For $k = -1$, $\dot{R}^2$ is always positive and $R \to t$ as $t \to \infty$. For $k = 0$, $\dot{R}^2$ goes to zero as $R \to \infty$. For $k = +1$, $\dot{R}^2$ drops to zero at $R_{\mathrm{max}} = (3M_{\mathrm{P}}^2/8\pi\rho)^{1/2}$ after which R begins decreasing. Thus $\Omega < 1$ corresponds to an open universe which will expand forever, $\Omega = 1$ is the critical or flat universe which will asymptotically expand to infinity while $\Omega > 1$ corresponds to a closed universe which will eventually recollapse.

Dynamical measurements of the present energy density in *all* gravitating matter require (see Peebles 1993, Dekel 1994)

$$\Omega_0 \approx 0.1 - 1 \ , \tag{2.22}$$

although such techniques are insensitive to matter which is not clustered on the largest scales probed (for example relativistic particles). The present energy density of visible radiation alone is better known, since it is dominated by that of the blackbody CMB with present temperature (Mather *et al* 1994)

$$T_0 = 2.726 \pm 0.01\,\mathrm{K}, \tag{2.23}$$

hence, defining $\Theta \equiv T_0/2.73\,\mathrm{K}$,

$$\rho_{\gamma_0} = \frac{\pi^2 T_0^4}{15} = 2.02 \times 10^{-51} \Theta^4 \ \mathrm{GeV}^4, \tag{2.24}$$

and

$$\Omega_{\gamma_0} = \frac{\rho_{\gamma_0}}{\rho_{c_0}} = 2.49 \times 10^{-5} \Theta^4 h^{-2}. \tag{2.25}$$

A primordial background of (three) massless neutrinos is also believed to be present (see (2.71)); this raises the total energy density in relativistic particles to

$$\Omega_{\mathrm{R}_0} = \Omega_{\gamma_0} + \Omega_{\nu_0} = 1.68\,\Omega_{\gamma_0} = 4.18 \times 10^{-5} \Theta^4 h^{-2}. \tag{2.26}$$

Since this is a negligible fraction of the total energy density Ω_0,† the universe is assumed to be matter dominated (MD) today by non-relativistic particles, i.e.

$$\Omega_0 \equiv \Omega_{\mathrm{R}_0} + \Omega_{\mathrm{NR}_0} \simeq \Omega_{\mathrm{NR}_0}. \tag{2.27}$$

In F-L-R-W models this has actually been true for most of the age of the universe, thus a lower bound to the age of the universe implies an upper bound on its matter content (see Weinberg 1972). Conservatively taking $t_0 > 10^{10}\,\mathrm{yr}$ and $h > 0.4$ requires (see Kolb and Turner 1990)

$$\Omega_{\mathrm{NR}_0} h^2 \lesssim 1 \quad \Rightarrow \quad \rho_{\mathrm{NR}_0} \lesssim 1.05 \times 10^{-5}\,\mathrm{GeV\,cm^{-3}} \ . \tag{2.28}$$

However as R decreases, ρ_{R} rises faster than ρ_{NR} so that the universe would have been radiation dominated (RD) by relativistic particles for

$$\frac{R}{R_0} < \frac{R_{\mathrm{EQ}}}{R_0} = 4.18 \times 10^{-5} \Theta^4 \ (\Omega_0 h^2)^{-1}. \tag{2.29}$$

† There can be a much higher energy density in massless particles such as neutrinos or hypothetical Goldstone bosons (see Kolb 1980) if these have been created relatively recently rather than being relics of the early universe.

Assuming that the expansion is adiabatic, the scale-factor is related to the blackbody photon temperature $T\,(\equiv\ T_\gamma)$ as $RT\ =$ constant (see (2.47)). Hence 'radiation' overwhelmed 'matter' for

$$T > T_{\rm EQ} = 5.63 \times 10^{-9}\,{\rm GeV}\ (\Omega_0 h^2 \Theta^{-3})\ . \tag{2.30}$$

Cosmological processes of interest to particle physics therefore occured during the RD era, with which we will be mainly concerned in subsequent sections.

2.2. Thermal history of the early universe

As the temperature rises, all particles are expected to ultimately achieve thermodynamic equilibrium through rapid interactions, facilitated by the increasing density. The interaction rate Γ typically rises much faster with temperature than the expansion rate H, hence the epoch at which Γ equals H is usually taken to mark the onset of equilibrium (see Wagoner 1980). More precisely, kinetic equilibrium is established by sufficiently rapid elastic scattering processes, and chemical equilibrium by processes which can create and destroy particles. Fortunately the particle densities do not usually become high enough for many-body interactions to be important and the interaction strengths remain in the perturbative domain, particularly because of asymptotic freedom for the strong interactions. Hence the approximation of an ideal gas (see Landau and Lifshitz 1982) is usually a good one, except near phase transitions. This vastly simplifies the thermodynamics of the radiation-dominated (RD) era.

Matters become complicated at temperatures much higher than the masses of the particles involved, since the cross-section for $2 \rightarrow 2$ processes ultimately decreases $\propto T^{-2}$ on dimensional grounds, hence $\Gamma\,(\propto T)$ then falls behind $H\,(\propto T^2)$ at some critical temperature (Ellis and Steigman 1979). Moreover, at temperatures approaching the Planck scale, the shrinking causal horizon imposes a lower cutoff on the energies of particles (Ellis and Steigman 1979), while the number of particles in any locally flat region of space-time becomes negligible (Padmanabhan and Vasanti 1982). Enqvist and Eskola (1990) have performed a computer simulation to study the relaxation of a weakly interacting relativistic gas with an initially non-thermal momentum distribution towards thermal equilibrium in the early universe. They find that kinetic equlibrium is achieved after only a few $2 \rightarrow 2$ elastic collisions, while chemical equilibrium takes rather longer to be established through $2 \rightarrow 3$ number-*changing* processes. In the extreme case that the universe is created as an initially cold gas of particles at the Planck scale (e.g. by quantum fluctuations), elastic scatterings achieve a (maximum) temperature of $\approx 3 \times 10^{14}\,{\rm GeV}$ while chemical equilibrium is only established at $\approx 10^{12}\,{\rm GeV}$, i.e. well below the grand unification scale (see also Elmfors *et al* 1994). For the QCD gas in particular, the annihilation rate for quarks to gluons falls behind H at $\approx 3 \times 10^{14}\,{\rm GeV}$, above which chemical equilibrium is not achieved (Enqvist and Sirkka 1993).

For an ideal gas, the equilibrium phase space density of particle type i is

$$f_i^{\rm eq}(q,T) = \left[\exp\left(\frac{E_i - \mu_i}{T}\right) \mp 1\right]^{-1} , \tag{2.31}$$

where $E_i \equiv \sqrt{m_i^2 + q^2}$, $-/+$ refers to Bose-Einstein/Fermi-Dirac statistics and μ_i is a possible chemical potential. The chemical potential is additively conserved in all reactions. Hence it is zero for particles such as photons and Z^0 bosons which can be emitted or absorbed in any number (at high enough temperatures) † and consequently equal and opposite for a particle and its antiparticle, which can annihilate into such gauge bosons. A finite *net* chemical potential for any species therefore corresponds to a particle-antiparticle asymmetry, i.e. a non-zero value for any associated conserved quantum number. Empirically, the net electrical charge of the universe is consistent with zero and the net baryon number is quite negligible relative to the number of photons: $(N_B - N_{\bar{B}})/N_\gamma \lesssim 10^{-9}$ (see Steigman 1976). Hence for most purposes it is reasonable to set μ_e and μ_B to be zero. The net lepton number is presumably of the same order as the baryon number so we can consider μ_ν to be zero for all flavours of (massless) neutrinos as well. However if the baryon minus lepton number $(B - L)$ is not zero, there may well be a large chemical potential in neutrinos which can influence nucleosynthesis (see section 3.3). (Also, even a small asymmetry, comparable to that observed in baryons, may enable a similarly *massive* particle (see section 5.2) to contribute significantly to the energy density of the universe.)

The thermodynamic observables number density, energy density and pressure, in equilibrium, are then functions of the temperature alone (see Harrison 1973):

$$\begin{aligned} n_i^{\rm eq}(T) &= g_i \int f_i^{\rm eq}(q,T)\,\frac{{\rm d}^3 q}{(2\pi)^3} = \frac{g_i}{2\pi^2} T^3 I_i^{11}(\mp) , \\ \rho_i^{\rm eq}(T) &= g_i \int E_i(q)\, f_i^{\rm eq}(q,T)\,\frac{{\rm d}^3 q}{(2\pi)^3} = \frac{g_i}{2\pi^2} T^4 I_i^{21}(\mp) , \\ p_i^{\rm eq}(T) &= g_i \int \frac{q^2}{3E_i(q)}\, f_i^{\rm eq}(q,T)\,\frac{{\rm d}^3 q}{(2\pi)^3} = \frac{g_i}{6\pi^2}\, T^4 I_i^{03}(\mp) , \end{aligned} \tag{2.32}$$

where,

$$I_i^{mn}(\mp) \equiv \int_{x_i}^{\infty} y^m (y^2 - x_i^2)^{n/2}\, ({\rm e}^y \mp 1)^{-1} dy , \qquad x_i \equiv \frac{m_i}{T} , \tag{2.33}$$

g_i is the number of internal (spin) degrees of freedom, and $-/+$ refers as before to bosons/fermions. These equations yield the relation

$$\frac{{\rm d}p^{\rm eq}}{{\rm d}T} = \frac{(\rho^{\rm eq} + p^{\rm eq})}{T} , \tag{2.34}$$

† This need not be true for $W^\pm$ bosons and gluons which carry non-trivial quantum numbers. We must *assume* that the universe has no net colour or hypercharge (see Haber and Weldon 1981).

which is just the second law of thermodynamics (see Weinberg 1972).

For relativistic (R) particles with $x \ll 1$, the integrals (2.33) are

$$\begin{aligned} \text{bosons}: \quad & I_{\rm R}^{11}(-) = 2\zeta(3) , \qquad I_{\rm R}^{21}(-) = I_{\rm R}^{03}(-) = \frac{\pi^4}{15} , \\ \text{fermions}: \quad & I_{\rm R}^{11}(+) = \frac{3\zeta(3)}{2} , \qquad I_{\rm R}^{21}(+) = I_{\rm R}^{03}(+) = \frac{7\pi^4}{120} , \end{aligned} \tag{2.35}$$

where ζ is the Riemann Zeta function and $\zeta(3) = 1.202$; for example, photons with $g_\gamma = 2$ have $n_\gamma = \frac{2\zeta(3)}{\pi^2}T^3$ and $\rho_\gamma = 3p_\gamma = \frac{\pi^2}{15}T^4$. (Since photons are always in equilibrium at these epochs, and indeed *define* the temperature T, we will not bother with the superscript eq for n_γ, ρ_γ or p_γ.)

For non-relativistic (NR) particles, which have $x \gg 1$, we recover the Boltzmann distribution

$$n_{\rm NR}^{\rm eq}(T) = \frac{\rho_{\rm NR}^{\rm eq}(T)}{m} = \frac{g}{(2\pi)^{3/2}} T^3 x^{3/2} {\rm e}^{-x}, \qquad p_{\rm NR} \simeq 0 , \tag{2.36}$$

independently of whether the particle is a boson or fermion. Non-relativistic particles, of course, contribute negligibly to the energy density in the RD era. It should be noted that the Boltzmann distribution is *not* invariant under the cosmic expansion, hence non-relativistic particles can maintain equilibrium only if they interact rapidly with a (dominant) population of relativistic particles (see Bernstein 1988).

It is then convenient to parametrize:

$$\rho_i^{\rm eq}(T) \equiv \left(\frac{g_{\rho_i}}{2}\right) \rho_\gamma , \qquad \text{i.e.} \quad g_{\rho_i} = \frac{15}{\pi^4} g_i I_i^{21}(\mp) , \tag{2.37}$$

so that g_{ρ_i} equals g_i for a relativistic boson, $\frac{7}{8}g_i$ for a relativistic fermion, and is negligibly small ($< 10\%$ correction) for a non-relativistic particle. When all particles present are in equilibrium through rapid interactions, the total number of relativistic degrees of freedom is thus given by summing over all interacting relativistic bosons (B) and fermions (F):

$$g_{\rm R} = \sum_{\rm B} g_i + \frac{7}{8} \sum_{\rm F} g_i . \tag{2.38}$$

At any given time, not all particles will, in fact, be in equilibrium at a common temperature T. A particle will be in kinetic equilibrium with the background thermal plasma (i.e. $T_i = T$) only while it is interacting, i.e. as long as the scattering rate,

$$\Gamma_{\rm scat} = n_{\rm scat} \langle \sigma_{\rm scat} v \rangle , \tag{2.39}$$

exceeds the expansion rate H. Here $\langle \sigma_{\rm scat} v \rangle$ is the (velocity averaged) cross-section for $2 \to 2$ processes such as $i\gamma \to i\gamma$ and $i\ell^\pm \to i\ell^\pm$ which maintain good thermal contact between the i particles and the particles (of density $n_{\rm scat}$) constituting the background plasma. (ℓ refers in particular to electrons which are abundant down to $T \sim m_{\rm e}$ and remain strongly coupled to photons via Compton scattering through the entire RD era, so that $T_{\rm e} = T$ always.) The i particle is said to 'decouple' at $T = T_{\rm D}$ when the condition

$$\Gamma_{\rm scat}(T_{\rm D}) \simeq H(T_{\rm D}) \tag{2.40}$$

is satisfied. Of course no particle is ever truly decoupled since there are always *some* residual interactions; however such effects are calculable (e.g. Dodelson and Turner (1992) and are generally negligible.

If the particle is relativistic at this time (i.e. $m_i < T_{\rm D}$), then it will also have been in chemical equilibrium with the thermal plasma (i.e. $\mu_i + \mu_{\bar{i}} = \mu_{\ell^+} + \mu_{\ell^-} = \mu_\gamma = 0$) through processes such as $i\bar{i} \leftrightarrow \gamma\gamma$ and $i\bar{i} \leftrightarrow \ell^+\ell^-$.† Hence its abundance at decoupling will be just the equilibrium value

$$n_i^{\rm eq}(T_{\rm D}) = \left(\frac{g_i}{2}\right) n_\gamma(T_{\rm D})\, f_{\rm B,F} \ , \tag{2.41}$$

where $f_{\rm B} = 1$ and $f_{\rm F} = \frac{3}{4}$ corresponding to whether i is a boson or a fermion.

Subsequently, the decoupled i particles will expand freely without interactions so that their number in a *comoving* volume is conserved and their pressure and energy density are functions of the scale-factor R *alone*. Although non-interacting, their phase space distribution will retain the equilibrium form (2.31), with T substituted by T_i, as long as the particles remain *relativistic*, which ensures that both E_i and T_i scale as R^{-1}. Initially, the temperature T_i will continue to track the photon temperature T. Now as the universe cools below various mass thresholds, the corresponding massive particles will become non-relativistic and annihilate. (For massive particles in the Standard Model, such annihilation will be almost total since all such particles have strong and/or electromagnetic interactions.) This will heat the photons and other interacting particles, but not the decoupled i particles, so that T_i will now drop below T and, consequently, n_i/n_γ will decrease below its value at decoupling.

To calculate this it is convenient, following Alpher *et al* (1953), to divide the total pressure and energy density into interacting (I) and decoupled (D) parts, which are, respectively, functions of T and R alone:

$$p = p_{\rm I}(T) + p_{\rm D}(R) \ , \qquad \rho = \rho_{\rm I}(T) + \rho_{\rm D}(R) \ . \tag{2.42}$$

The conservation equation (2.7) written as

$$R^3 \frac{{\rm d}p}{{\rm d}T} = \frac{\rm d}{{\rm d}T}\left[R^3(\rho + p)\right] \tag{2.43}$$

† In fact, neutrinos, which are both massless and weakly interacting, are the only particles in the Standard Model which satisfy this condition. The other particles, being both massive and strongly and/or electromagnetically interacting, would have self-annihilated when they became non-relativistic and would therefore not have survived with any appreciable abundance until the epoch of kinetic decoupling which generally occurs much later.

then reduces to

$$\frac{\mathrm{d}\ln R}{\mathrm{d}\ln T} = -\frac{1}{3}\frac{(\mathrm{d}\rho_\mathrm{I}/\mathrm{d}\ln T)}{(\rho_\mathrm{I}+p_\mathrm{I})} \ , \tag{2.44}$$

upon requiring the number conservation of decoupled particles ($n_\mathrm{D}R^3 = $ constant) and neglecting the pressure of non-relativistic decoupled particles. Combining with the second law of thermodynamics (2.34), we obtain

$$\frac{\mathrm{d}\ln R}{\mathrm{d}\ln T} = -1 - \frac{1}{3}\frac{\mathrm{d}\ln\left(\frac{\rho_\mathrm{I}+p_\mathrm{I}}{T^4}\right)}{\mathrm{d}\ln T} \ , \tag{2.45}$$

which integrates to,

$$\ln R = -\ln T - \frac{1}{3}\ln\left(\frac{\rho_\mathrm{I}+p_\mathrm{I}}{T^4}\right) + \text{constant}\,. \tag{2.46}$$

If $(\rho_\mathrm{I}+p_\mathrm{I})/T^4$ is constant, as for a gas of blackbody photons, this yields the adiabatic invariant

$$RT = \text{constant} \tag{2.47}$$

which we have used earlier to obtain (2.30). The second term on the RHS of (2.46) is a correction which accounts for departures from adiabaticity due to changes in the number of interacting species.

(Another possible source of non-adiabaticity is a phase transition which may release latent heat thus increasing the entropy. The ideal gas approximation is then no longer applicable and finite temperature field theory must be used (see Bailin and Love 1986, Kapusta 1988). The standard cosmology assumes parenthetically that such phase transitions occurred rapidly at their appropriate critical temperature, generating negligible latent heat, i.e. that they were second-order. However, phase transitions associated with spontaneous symmetry breaking in gauge theories may well be first-order; this possibility is in fact exploited in the inflationary universe model (Guth 1981, see Linde 1990) to account for the observed large entropy content of the universe, as mentioned earlier. We will shortly discuss the possible generation of entropy during the quark-hadron and electroweak phase transitions.)

Epochs where the number of interacting species is different can now be related by noting that (2.45) implies the constancy of the specific entropy, S_I, in a comoving volume:

$$\frac{\mathrm{d}S_\mathrm{I}}{\mathrm{d}T} = 0 \ , \qquad S_\mathrm{I} \equiv s_\mathrm{I}R^3, \tag{2.48}$$

Here, s_I, the specific entropy density, sums over all *interacting* species in equilibrium:

$$s_\mathrm{I} \equiv \frac{\rho_\mathrm{I}+p_\mathrm{I}}{T} = \sum_\mathrm{int} s_i \ , \tag{2.49}$$

where, using (2.32),

$$s_i(T) = g_i \int \frac{3m_i^2 + 4q^2}{3E_i(q)\,T}\, f_i^{\rm eq}(q,T)\,\frac{{\rm d}^3 q}{(2\pi)^3}\ . \tag{2.50}$$

As with the energy density (2.37), we can conveniently parametrize the entropy density of particle i in terms of that for photons:

$$s_i(T) \equiv \left(\frac{g_{s_i}}{2}\right)\left(\frac{4}{3}\frac{\rho_\gamma}{T}\right)\ , \tag{2.51}$$

i.e.

$$g_{s_i} = \frac{45}{4\pi^4} g_i \left[I_i^{21}(\mp) + \frac{1}{3} I_i^{03}(\mp) \right], \tag{2.52}$$

so defined that g_{s_i} (like g_{ρ_i}) equals g_i for a relativistic boson, $\frac{7}{8}g_i$ for a relativistic fermion, and is negligibly small for a non-relativistic particle. Hence the number of *interacting* degrees of freedom contributing to the specific entropy density is given by

$$g_{s_{\rm I}} \equiv \frac{45}{2\pi^2}\frac{s_{\rm I}}{T^3} = \sum_{\rm int} g_{s_i}\ . \tag{2.53}$$

This is, of course, the same as $g_{\rm R}$ (2.38) when all particles are relativistic. (This parameter has been variously called $g_{\rm I}$ (Steigman 1979), $g_{\rm E}$ (Wagoner 1980) and g' (Olive *et al* 1981a) in the literature.)

It is now simple to calculate how the temperature of a particle i which decoupled at $T_{\rm D}$ relates to the photon temperature T at a later epoch. For $T < T_{\rm D}$, the entropy in the decoupled i particles and the entropy in the still interacting j particles are *separately* conserved:

$$\begin{aligned} S - S_{\rm I} = s_i\,R^3 \quad &= \frac{2\pi^2}{45} g_{s_i}(T)\,(R\,T)_i^3, \\ S_{\rm I} = \sum_{j\neq i} s_j(T)\,R^3 &= \frac{2\pi^2}{45} g_{s_{\rm I}}(T)\,(R\,T)^3\ , \end{aligned} \tag{2.54}$$

where S is the conserved total entropy at $T > T_{\rm D}$. Given that $T_i = T$ at decoupling, this then yields for the subsequent ratio of temperatures (Srednicki *et al* 1988, Gondolo and Gelmini 1991):

$$\frac{T_i}{T} = \left[\frac{g_{s_i}(T_{\rm D})}{g_{s_i}(T)} \frac{g_{s_{\rm I}}(T)}{g_{s_{\rm I}}(T_{\rm D})} \right]^{1/3}\ . \tag{2.55}$$

Note the difference from the expression $T_i/T = [g_{s_{\rm I}}(T)/g_{s_{\rm I}}(T_{\rm D})]^{1/3}$ given by Olive *et al* (1981a), which is not always correct, for example when the decoupled particles have new interactions which allow them to subsequently annihilate into other non-interacting particles, thus changing g_{s_i} from its value at decoupling (e.g. Kolb *et al* 1986c).

The degrees of freedom specifying the conserved total entropy is then given, following decoupling, by

$$g_s(T) \equiv \frac{45}{2\pi^2}\frac{S}{T^3R^3} = g_{s_{\rm I}}(T)\left[1 + \frac{g_{s_i}(T_{\rm D})}{g_{s_{\rm I}}(T_{\rm D})}\right] . \tag{2.56}$$

When the species i becomes non-relativistic and annihilates into the other relativistic interacting particles before decoupling, the few remaining decoupled particles have negligible entropy content, hence $g_{s_i}(T_{\rm D}) \simeq 0$. Then g_s just counts all *interacting* species at temperature T which have now acquired the entropy released by the annihilations, i.e. $g_s \simeq g_{s_{\rm I}}$ (2.53). However when the decoupled species is relativistic and carries off its own entropy which is *separately* conserved, then g_s explicitly includes its contribution to the conserved total entropy, by weighting appropriately by its temperature, which may now be smaller (according to (2.55)) than the photon temperature T:

$$\begin{aligned} g_s(T) &= \sum_{j\neq i} g_{s_j}(T) + g_{s_i}(T_i)\left(\frac{T_i}{T}\right)^3 \\ &\simeq \sum_{\rm B} g_i\left(\frac{T_i}{T}\right)^3 + \frac{7}{8}\sum_{\rm F} g_i\left(\frac{T_i}{T}\right)^3 . \end{aligned} \tag{2.57}$$

The last equality follows when all particles are relativistic. (This parameter is called $g_{\star s}$ by Scherrer and Turner (1986) and Kolb and Turner (1990), h by Srednicki *et al* (1988) and $h_{\rm eff}$ by Gondolo and Gelmini (1991).) If several different species decouple while still relativistic, as is possible in extensions of the Standard Model which contain new weakly interacting massless particles, then (2.56) is easily generalized to (Gondolo and Gelmini 1991)

$$g_s(T) = g_{s_{\rm I}}(T)\prod_{i{\rm dec}}\left[1 + \frac{g_{s_i}(T_{{\rm D}i})}{g_{s_{\rm I}}(T_{{\rm D}i})}\right] . \tag{2.58}$$

We now have an useful fiducial in the total entropy density,

$$s(T) \equiv \frac{2\pi^2}{45} g_s(T)T^3 , \tag{2.59}$$

which *always* scales as R^{-3} by appropriately keeping track of any changes in the number of degrees of freedom. Therefore the ratio of the decoupled particle density to the blackbody photon density is subsequently related to its value at decoupling as:

$$\frac{(n_i/n_\gamma)_T}{(n_i^{\rm eq}/n_\gamma)_{T_{\rm D}}} = \frac{g_s(T)}{g_s(T_{\rm D})} = \frac{N_\gamma(T_{\rm D})}{N_\gamma(T)} , \tag{2.60}$$

where $N_\gamma = R^3\, n_\gamma$ is the total number of blackbody photons in a comoving volume.

The total energy density may be similarly parametrized as:

$$\rho(T) = \sum \rho_i^{\rm eq} \equiv \left(\frac{g_\rho}{2}\right)\rho_\gamma = \frac{\pi^2}{30} g_\rho T^4 , \tag{2.61}$$

i.e.

$$\begin{aligned} g_\rho &= \sum_{j\neq i} g_{\rho_j}(T) + g_{\rho_i}(T_i)\left(\frac{T_i}{T}\right)^4 \\ &\simeq \sum_{\rm B} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_{\rm F} g_i \left(\frac{T_i}{T}\right)^4 , \end{aligned} \tag{2.62}$$

where the last equality follows when all particles are relativistic. (The parameter g_ρ is called g by Steigman (1979), Olive *et al* (1981a) and Srednicki *et al* (1988) and $g_{\rm eff}$ by Gondolo and Gelmini (1991); more often (e.g. Wagoner 1980, Scherrer and Turner 1986, Kolb and Turner 1990) it is called $g_\star$.)

Let us now rewrite (2.45) more compactly as

$$\frac{{\rm d}R}{R} = -\frac{{\rm d}T}{T} - \frac{1}{3}\frac{{\rm d}g_{s_{\rm I}}}{g_{s_{\rm I}}}. \tag{2.63}$$

using (2.53). (This expression is also given by Srednicki *et al* (1988), however with g_s rather than $g_{s_{\rm I}}$ on the RHS; admittedly this makes no difference in practice.) Using this, we can now obtain the relationship between the time t and the temperature T by integrating the F-L equation (2.4). Since the curvature term k/R^2 is negligible during the RD era, we have

$$H = \sqrt{\frac{8\pi\,\rho}{3\,M_{\rm P}^2}} = 1.66\ g_\rho^{1/2}\frac{T^2}{M_{\rm P}}\ , \tag{2.64}$$

and,

$$\begin{aligned} t &= \int \left(\frac{3\,M_{\rm P}^2}{8\pi\,\rho}\right)^{1/2}\frac{{\rm d}R}{R} \\ &= -\int \left(\frac{45\,M_{\rm P}^2}{4\pi^3}\right)^{1/2} g_\rho^{-1/2}\left(1+\frac{1}{3}\frac{{\rm d}\ln g_{s_{\rm I}}}{{\rm d}\ln T}\right)\frac{{\rm d}T}{T^3}. \end{aligned} \tag{2.65}$$

During the periods when ${\rm d}g_{s_{\rm I}}/{\rm d}T \simeq 0$, i.e. away from mass thresholds and phase transitions, this yields the useful commonly used approximation

$$t = \left(\frac{3\,M_{\rm P}^2}{32\pi\,\rho}\right)^{1/2} = 2.42\ g_\rho^{-1/2}\left(\frac{T}{\rm MeV}\right)^{-2}\ {\rm sec}\,. \tag{2.66}$$

The above discussion is usually illustrated by the example of the decoupling of massless neutrinos in the Standard Model. Taking the thermally-averaged cross-section to be $\langle\sigma v\rangle \sim G_{\rm F}^2E^2 \sim G_{\rm F}^2T^2$, the interaction rate is $\Gamma = n\langle\sigma v\rangle \sim G_{\rm F}^2T^5$ (since $n \approx T^3$). This equals the expansion rate $H \sim T^2/M_{\rm P}$ at the decoupling temperature

$$T_{\rm D}(\nu) \sim (G_{\rm F}^2M_{\rm P})^{-1/3} \sim 1\,{\rm MeV}\,. \tag{2.67}$$

(A more careful estimate of $\langle\sigma_{\nu\bar\nu\to{\rm e^+e^-}}v\rangle$ (Dicus *et al* 1982, Enqvist *et al* 1992a) gives $T_{\rm D}(\nu_\mu,\,\nu_\tau) = 3.5$ MeV for the neutral current interaction and $T_{\rm D}(\nu_{\rm e}) = 2.3$ MeV, upon adding the charged current interaction.) At this time $n_\nu^{\rm eq} = \frac{3}{4} n_\gamma$ since $T_\nu = T$ and $g_\nu = 2$. (In the Standard Model, right-handed neutrinos transform as singlets of $SU(2)_{\rm L} \otimes U(1)_Y$ and have no gauge interactions, hence these states cannot be excited thermally unless Dirac masses are introduced (see section 5.1.1).) Subsequently as T drops below the electron mass $m_{\rm e}$, the electrons and positrons annihilate (almost) totally, heating the photons but not the decoupled neutrinos. From (2.55) we see that while g_ν does not change following decoupling, the number of *other* interacting degrees of freedom decreases from 11/2 (γ and $\rm e^\pm$) to 2 (γ only), hence the comoving number of blackbody photons increases by the factor

$$\frac{N_\gamma\,(T \ll m_{\rm e})}{N_\gamma\,(T = T_{\rm D}(\nu))} = \left[\frac{(RT)_{T \ll m_{\rm e}}}{(RT)_{T=T_{\rm D}(\nu)}}\right]^3 = \frac{11}{4}\ , \tag{2.68}$$

so that subsequently

$$\left(\frac{n_\nu}{n_\gamma}\right)_{T \ll m_{\rm e}} = \frac{4}{11}\left(\frac{n_\nu^{\rm eq}}{n_\gamma}\right)_{T=T_{\rm D}(\nu)} = \frac{3}{11}\ . \tag{2.69}$$

The evolution of the neutrino temperature through the period of $\rm e^\pm$ annihilation can be computed using (2.52) and (2.55) (see Weinberg 1972):

$$\frac{T_\nu}{T} = \left(\frac{4}{11}\right)^{1/3}\left[1 + \frac{45}{2\pi^4}\left(I^{21}(+) + \frac{1}{3} I^{03}(+)\right)\right]^{1/3}\ . \tag{2.70}$$

The neutrinos remain relativistic and therefore continue to retain their equilibrium distribution function hence the degrees of freedom characterizing the present day entropy and energy densities are :

$$\begin{aligned} g_s\,(T \ll m_{\rm e}) &= g_\gamma + \frac{7}{8} N_\nu\, g_\nu \left(\frac{T_\nu}{T}\right)^3 = \frac{43}{11}\ , \\ g_\rho\,(T \ll m_{\rm e}) &= g_\gamma + \frac{7}{8} N_\nu\, g_\nu \left(\frac{T_\nu}{T}\right)^4 = 3.36\ , \end{aligned} \tag{2.71}$$

for 3 *massless* neutrino species (N_ν=3). Note that the increase in the number of comoving photons due to $\rm e^\pm$ annihilation (2.68) is indeed given, following (2.60), by the ratio $g_s(T_{\rm D}(\nu))/g_s(T_0) = \frac{43}{4}/\frac{43}{11} = \frac{11}{4}$.

Since neutrino decoupling occurs so close to $\rm e^+e^-$ annihilation, their residual interactions with the thermal plasma cause the neutrinos to be slightly heated by the resultant entropy release (Dicus *et al* 1982, Herrera and Hacyan 1989). This effect has been studied by Dolgov and Fukugita (1992) and, particularly carefully, by Dodelson and Turner (1992), who solve the governing Boltzmann equation with both scattering and annihilation processes included; Hannestad and Madsen (1995) have redone the exercise using Fermi-Dirac rather than Boltzmann statistics. The asymptotic energy density in electron neutrinos is found to be raised by 0.8% over the canonical estimate above, and that for muon and tau neutrinos by 0.4%, while the back reaction due to neutrino heating is found to suppress the increase in the comoving number of photons by 0.5%. These studies demonstrate that neutrino decoupling is not an instantaneous process, particularly since the interaction cross-section increases with the neutrino energy. Consequently the spectrum of the decoupled neutrinos deviates slightly from the Fermi-Dirac form, causing the effective neutrino temperature ($\equiv -q/\ln f_\nu(q,t)$) to increase with momentum. The increase is however only by 0.7% even at relatively high momenta, $q/T \approx 10$, justifying the usual approximation of instantaneous decoupling.

The detailed formalism given above for reconstructing the thermal history of the RD era is essential for accurately calculating the abundances of hypothetical massive particles or massless particles with unusual interactions, which may affect BBN. For the moment we restrict our attention to the Standard $SU(3)_{\rm c}\otimes SU(2)_{\rm L}\otimes U(1)_Y$ Model and show in table 1 the temperature dependence of the number of interacting relativistic degrees of freedom, $g_{\rm R}(T)$ (2.38), as well as the factor $N_\gamma(T_0)/N_\gamma(T)$ (2.60) by which the comoving blackbody photon number is higher today, at $T = T_0$. In calculating $g_{\rm R}$ we have assumed that a massive particle remains relativistic down to $T \sim m_i$ and immediately annihilates completely into radiation, and that phase transitions happen instantaneously at the relevant critical temperature with negligible release of entropy; hence the quoted values are meaningful only when far away from mass thresholds and phase transitions. Apart from the massless neutrinos, all particles in the SM are strongly coupled to the thermal plasma while they are relativistic, hence g_s (2.57) equals g_ρ (2.62) and their common value equals $g_{\rm R}$, above the neutrino decoupling temperature $T_{\rm D}(\nu)$ of a few MeV, while their low temperature values are given in (2.71). Note that neutrino decoupling has no effect on the entropy or dynamics, hence g_s and g_ρ do not change (from their common value of 43/4 below the muon mass threshold) until $e^{\pm}$ annihilation occurs.

A more careful calculation has been done by Srednicki *et al* (1988) following a similar earlier exercise by Olive *et al* (1981a). By numerical integration over the phase-space density (using (2.37) and (2.52)), these authors obtain g_ρ and g_s as a continuous function of T rather than step-wise as in table 1; they also include the (small) contribution to the energy and entropy density from non-relativistic baryons and mesons. There is however considerable ambiguity concerning the thermodynamic history during the quark-hadron phase transition. As the critical temperature $T_{\rm c}^{\rm qh}$ is approached from below, particle interactions become important and the ideal gas approximation begins to break down; however at temperatures higher than $T_{\rm c}^* \approx 1$ GeV,† the asymptotic freedom of the strong interactions again permits the system to be decribed as an ideal gas of leptons, quarks and gauge bosons. Srednicki *et al* (1988) present curves for the behaviour of g_ρ and g_s in

† It is difficult to reliably calculate $T_{\rm c}^*$ because of non-perturbative effects in the strongly coupled quark-gluon plasma (see Shuryak 1980, Gross *et al* 1981).

the intervening region corresponding to two choices (150 and 400 MeV) of $T_{\rm c}^{\rm qh}$ and state that these bound the range of possibilities.‡ They also show the evolution of g_ρ during this epoch for the case when the phase transition is strongly first-order, a possibility suggested in the past by lattice gauge calculations which set quark masses to be zero (see Satz 1985, McLerran 1986). However recent lattice computations which have been performed with realistic masses for the u, d and s quarks suggest that there may be a second-order phase transition or even a 'cross-over' (see Toussaint 1992, Smilga 1995). (This is particularly important to keep in mind in the context of the bound imposed by BBN on superweakly interacting particles which may have decoupled during this era.) In figure 2 we show both g_ρ and g_s as a function of temperature as computed by Srednicki *et al* (1988). As we have emphasized, these curves are only meant to indicate the range of possibilities in this temperature region.

The last three entries in table 1 are uncertain because of our ignorance about the mass of the Higgs boson which is responsible for $SU(2)\otimes U(1)$ symmetry breaking. It has been assumed here that the Higgs is sufficiently heavy that the electroweak phase transition is effectively second-order and occurs at a critical temperature (see Linde 1979, Weinberg 1980)

$$T_{\rm c}^{\rm EW} \simeq 300\,{\rm GeV}\left[1+\left(\frac{m_{H^0}}{150\,{\rm GeV}}\right)^{-2}\right]^{-1/2}. \qquad (2.72)$$

Coleman and Weinberg (1973) had studied the possibility that the Higgs is *massless* at tree-level so that the $SU(2)\otimes U(1)$ symmetry is classically scale-invariant and broken only by radiative corrections. These corrections generate a small mass, $m_{H^0} \simeq 10\,{\rm GeV}$ (for a light top quark); the critical temperature is then $T_{\rm c}^{\rm EW} \simeq 25\,{\rm GeV}$ and the phase transition is strongly first-order, generating a large and probably unacceptable amount of entropy (see Sher 1989). However the Coleman-Weinberg theory is untenable if the top quark mass exceeds 85 GeV as is now established by its recent detection at *Fermilab* with $m_t = 180 \pm 12\,{\rm GeV}$ and, further, such a light Higgs is now ruled out by *LEP* which sets the bound $m_{H^0} > 60\,{\rm GeV}$ (Particle Data Group 1996).

‡ Srednicki *et al* did not provide any quantitative details as to how these curves are obtained. It appears (K A Olive, private communication) that these authors adopted the naïve thermodynamic picture (see Olive 1990b) in which a hadron is viewed as a 'bag' containing quarks and gluons so that the pressure and energy density in the region of interest ($T \sim 100 - 1000$ MeV) are taken to be $P = \frac{\pi^2}{90}[2(N_{\rm c}^2-1)+\frac{7}{2}N_{\rm c}N_{\rm fl}]T^4 - B, \rho = 3P + 4B$, where $N_{\rm c}$ (=3) is the number of colours, $N_{\rm fl}$ (=3) is the number of light quark flavours (u, d, s), and B is the bag constant representing the vacuum energy difference between the two phases (which essentially determines $T_{\rm c}^{\rm qh}$). In this picture the pressure in the quark-gluon phase drops steeply with temperature during 'confinement', which occurs at a higher temperature for a higher adopted value of B. The pressure in the hadronic phase at lower temperatures (calculated assuming non-interacting particles) is approximately constant hence phase equilibrium is achieved when the pressure in the two phases become equal at $T \approx 100$ MeV.

Recently, cosmological electroweak symmetry breaking has come under renewed scrutiny following the realization that fermion-number violating transitions are unsuppressed at this epoch; the possibility of generating the baryon asymmetry of the universe then arises if the neccessary non-equilibrium conditions can be achieved via a first-order phase transition (see Cohen *et al* 1994, Rubakov and Shaposhnikov 1996). While this topic is outside the scope of the present review, we note that according to the detailed studies (see Romão and Freire 1994, Yaffe 1995), the phase transition is at best *very* weakly first-order. Hence our assumption that any generation of entropy is insignificant is probably justified although in extensions of the SM where the Higgs sector is enlarged, e.g. in supersymmetric models, the phase transition may well be strongly first-order with substantial entropy generation (e.g. Brignole *et al* 1994). These conclusions are however based on perturbation theory. Recent non-perturbative studies of the $SU(2)$ Higgs model in three dimensions, using both analytic techniques (e.g. Buchmüller and Philipsen 1995) and lattice simulations (e.g. Kajantie *et al* 1996) show that for large Higgs mass there will be no phase transition but rather a 'cross-over' since there is no gauge-invariant order parameter. Recent lattice calculations (e.g. Boyd *et al* 1995, Philipsen *et al* 1996) also indicate the presence of bound states in the plasma at high temperatures due to the non-abelian nature of the colour and electroweak forces; however the consequent departure from ideal gas behaviour is only of $\mathcal{O}(10\%)$.

At even higher temperatures, g_R will depend on the adopted theory. For example, in the minimal $SU(5)$ GUT, with three families of fermions and a single (complex) **5** of Higgs plus a **24** adjoint of Higgs to break $SU(5)$, the number of degrees of freedom above the unification scale is given by:

$$g_\mathrm{R}\ (T\ \gtrsim\ M_\mathrm{GUT})_{SU(5)} = (2\times 24 + 24 + 2\times 5) + \tfrac{7}{8}\ (2\times 3\times 15) = \tfrac{647}{4}\ . \tag{2.73}$$

In a supersymmetric model below the SUSY-breaking scale, the degrees of freedom would at least double overall; in the minimal supersymmetric Standard Model (MSSM) with three families of fermions and two (complex) doublets of Higgs plus all the superpartners,

$$g_\mathrm{R}\ (T\ \lesssim\ M_\mathrm{SUSY})_\mathrm{MSSM} = (24+8+90)\,(1+\tfrac{7}{8}) = \tfrac{915}{4}\ , \tag{2.74}$$

when all particles are relativistic. The present experimental limits (Particle Data Group 1996) allow some supersymmetric particles, if they exist, to be light enough to possibly affect the last few entries in table 1. Of course given the mass spectrum of any specific supersymmetric (or other) model, table 1 can be recalculated accordingly.

To summarize, although the formulation of kinetic theory in the expanding universe is far from trivial (see Bernstein 1988), the thermal history of the universe can be reconstructed fairly reliably back to the Fermi scale, and, with some caveats, nearly upto the GUT scale. This is possible primarily because we are dealing with a dilute radiation dominated plasma in which non-relativistic particles have negligible abundances and are forced to remain in equilibrium with the relativistic particles. Interaction rates typically rise faster with temperature than the expansion rate of the universe, justifying the usual assumption of equilibrium. The major uncertainties arise where the ideal gas approximation breaks down, viz. at phase transitions associated with symmetry breaking, and at very high temperatures when equilibrium may not be achieved.

3. Primordial nucleosynthesis

We now turn to the creation of the light elements towards the end of the "first three minutes" which provides the deepest detailed probe

of the Big Bang.† The physical processes involved have been well understood for some time (Hayashi 1950, Alpher *et al* 1953, Hoyle and Tayler 1964, Peebles 1966a,b, Wagoner *et al* 1967) ‡ and the final abundances of the synthesized elements are sensitive to a variety of parameters and physical constants. This enables many interesting constraints to be derived on the properties of relic particles or new physics which may influence BBN and alter the synthesized abundances. It must, of course, first be demonstrated that the expected elemental abundances in the standard BBN model are consistent with observations. There is a complication here in that the light elements are also created and destroyed in astrophysical environments so their abundances today differ significantly from their primordial values. The latter can only be inferred after correcting for the complex effects of galactic chemical evolution (see Tinsley 1980) over several thousand million years and this necessarily introduces uncertainties in the comparison with theory.

There are many excellent reviews of both theoretical and observational aspects of BBN (see Peebles 1971, Weinberg 1972, Schramm and Wagoner 1977, Boesgaard and Steigman 1985, Pagel 1992, Reeves 1994). However these usually quote results obtained by numerical means, while in order to appreciate the reliability (or otherwise!) of constraints derived therein one first requires a good analytic understanding of the physical processes involved. Secondly, as noted above, the observed elemental abundances have to be corrected for evolutionary effects and there are differences in the approaches taken by different authors in inferring the primordial values. It is

† The temperature fluctuations in the CMB observed (Smoot *et al* 1992) by the *Cosmic Background Explorer* (*COBE*) very probably reflect physical conditions at a much earlier epoch, if for example these are due to quantum perturbations generated during inflation (see Linde 1990). This interpretation is however not sufficiently firmly established as yet to provide a reliable "laboratory" for particle physics, although this may well happen as further observational tests are performed (see Steinhardt 1995).

‡ Gamow and collaborators pioneered such calculations in the 1940s (see Alpher and Herman 1950) but did not take into account the crucial role played by the weak interactions in maintaining neutron-proton equilibrium; for historical accounts see Alpher and Herman (1990) and Wagoner (1990).

therefore helpful to review the essential theory and the actual observational data before we examine the validity of the standard BBN model and then proceed to discuss the constraints imposed on new physics.

3.1. The standard BBN model

It is convenient to consider element synthesis in the early universe as occuring in two distinct stages: first the decoupling of the weak interactions which keep neutrons and protons in equilibrium, and second the onset, a little later, of the nuclear reactions which build up the light nuclei. It is possible to do this because the very high value of the entropy per nucleon ($s/n_{\rm N} \sim 10^{11}$) ensures that the equilibrium abundances of all bound nuclei are quite negligible as long as free nucleons are in equilibrium. We begin by outlining an elegant semi-analytic analysis of the first stage by Bernstein *et al* (1989) which follows the evolution of the neutron-to-proton ratio and allows the yield of ^{4}He, the primary product of BBN, to be calculated quite accurately *without* any detailed analysis of the nuclear reaction network. The latter is however neccessary to calculate the yields of less stable nuclei such as D, ^{3}He and ^{7}Li, which are the 'left over' products of nuclear burning. It is seen that the ^{4}He abundance depends sensitively on the Hubble expansion rate at this epoch (and therefore on the number of neutrino flavours) as well as on the neutron lifetime (which determines the rate of weak interactions), but only weakly on the nucleon density. Conversely, the abundances of the other light elements provide a sensitive probe of the nucleon density.

3.1.1. Neutron 'freeze-out': At sufficiently high temperatures (above a few MeV, as we shall see shortly) neutrons and protons are maintained in both kinetic equilibrium, i.e.

$$T_{\rm n} = T_{\rm p} = T_{\rm e} = T_{\nu_{\rm e}} = T \ , \tag{3.1}$$

and chemical equilibrium, i.e.

$$\mu_{\rm n} - \mu_{\rm p} = \mu_{\rm e^-} - \mu_{\nu_{\rm e}} = \mu_{\bar{\nu}_{\rm e}} - \mu_{\rm e^+} \ , \tag{3.2}$$

through the weak processes

$$\rm n + \nu_e \rightleftharpoons p + e^-, \quad n + e^+ \rightleftharpoons p + \bar{\nu}_e, \quad n \rightleftharpoons p + e^- + \bar{\nu}_e \ . \tag{3.3}$$

Defining $\lambda_{\rm np}$ as the summed rate of the reactions which convert neutrons to protons,

$$\lambda_{\rm np} = \lambda\ ({\rm n}\nu_{\rm e} \to {\rm pe}^-) + \lambda\ ({\rm ne}^+ \to {\rm p}\bar{\nu}_{\rm e}) + \lambda\ ({\rm n} \to {\rm pe}^-\bar{\nu}_{\rm e}) \ , \tag{3.4}$$

the rate $\lambda_{\rm pn}$ for the reverse reactions which convert protons to neutrons is given by detailed balance:

$$\lambda_{\rm pn} = \lambda_{\rm np}\ {\rm e}^{-\Delta m/T(t)}, \qquad \Delta m \equiv m_{\rm n} - m_{\rm p} = 1.293\,{\rm MeV} \,. \tag{3.5}$$

For the moment, we ignore the possibility of a large chemical potential in electron neutrinos which would otherwise appear in the exponent above (see (3.37)). The chemical potential of electrons is negligible since any excess of electrons which survives the annihilation epoch at $T \sim m_{\rm e}$ must equal the small observed excess of protons, given that the universe appears to be electrically neutral to high accuracy (Lyttleton and Bondi 1959, Sengupta and Pal 1996), i.e.

$$\frac{\mu_{\rm e}}{T} \approx \frac{n_{\rm e}}{n_\gamma} = \frac{n_{\rm p}}{n_\gamma} \sim 10^{-10}. \tag{3.6}$$

The evolution of the fractional neutron abundance $X_{\rm n}$ is described by the balance equation

$$\frac{{\rm d}X_{\rm n}(t)}{{\rm d}t} = \lambda_{\rm pn}(t)[1 - X_{\rm n}(t)] - \lambda_{\rm np}(t)X_{\rm n}(t) \ , \qquad X_{\rm n} \equiv \frac{n_{\rm n}}{n_{\rm N}} \ , \tag{3.7}$$

where $n_{\rm N}$ is the total nucleon density at this time, $n_{\rm N} = n_{\rm n} + n_{\rm p}$.† The equilibrium solution is obtained by setting ${\rm d}X_{\rm n}(t)/{\rm d}t = 0$:

$$X_{\rm n}^{\rm eq}(t) = \frac{\lambda_{\rm pn}(t)}{\Lambda(t)} = \left[1 + {\rm e}^{\Delta m/T(t)}\right]^{-1} \ , \qquad \Lambda \equiv \lambda_{\rm pn} + \lambda_{\rm np} \ , \tag{3.8}$$

while the general solution is

$$\begin{aligned} X_{\rm n}(t) &= \int_{t_{\rm i}}^{t} {\rm d}t' \ I(t,t') \ \lambda_{\rm pn}(t') \ + \ I(t,t_{\rm i}) \ X_{\rm n}(t_{\rm i}) \ , \\ I(t,t') &\equiv \exp\left[-\int_{t'}^{t} {\rm d}t'' \Lambda(t'')\right] \ . \end{aligned} \tag{3.9}$$

Since the rates $\lambda_{\rm pn}$ and $\lambda_{\rm np}$ are very large at early times, $I(t,t_{\rm i})$ will be negligible for a suitably early choice of the initial epoch $t_{\rm i}$, hence the initial value of the neutron abundance $X_{\rm n}(t_{\rm i})$ plays *no* role and thus does not depend on any particular model of the very early universe. For the same reason, $t_{\rm i}$ may be replaced by zero and the above expression simplifies to (Bernstein *et al* 1989)

$$\begin{aligned} X_{\rm n}(t) &= \int_0^t {\rm d}t' \ I(t,t') \ \lambda_{\rm pn}(t') \\ &= \frac{\lambda_{\rm pn}(t)}{\Lambda(t)} - \int_0^t {\rm d}t' \ I(t,t') \ \frac{\rm d}{{\rm d}t'}\left[\frac{\lambda_{\rm pn}(t')}{\Lambda(t')}\right]. \end{aligned} \tag{3.10}$$

Since the total reaction rate Λ is large compared to the rate of time variation of the individual rates, this can be written as

$$\begin{aligned} X_{\rm n}(t) &\simeq \frac{\lambda_{\rm pn}(t)}{\Lambda(t)} - \frac{1}{\Lambda(t)}\frac{\rm d}{{\rm d}t}\left[\frac{\lambda_{\rm pn}(t)}{\Lambda(t)}\right] \\ &\simeq X_{\rm n}^{\rm eq}\left[1 + \frac{H}{\Lambda}\frac{{\rm d}\ln X_{\rm n}^{\rm eq}}{{\rm d}\ln T}\right], \end{aligned} \tag{3.11}$$

† We will make a point of referring specifically to *nucleons* rather than to baryons as other authors do since there may well be other types of stable baryons, e.g. ‘strange quark nuggets’ (Witten 1984, see Alcock and Olinto 1988), which do not participate in nucleosynthesis.

using (3.8). Clearly, the neutron abundance tracks its value in equilibrium until the inelastic neutron-proton scattering rate Λ decreases sufficiently so as to become comparable to the Hubble expansion rate $H = \dot{R}/R \simeq -\dot{T}/T$. At this point the neutrons 'freeze-out', i.e. go out of *chemical* equilibrium, and subsequently, as we shall see, $X_{\rm n}$ relaxes to a constant value rather than following the exponentially falling value of $X_{\rm n}^{\rm eq}$. The freeze-out temperature can be approximately estimated by simply equating the expansion rate, $H \approx g_\rho^{1/2} T^2/M_{\rm P}$, to the reaction rate per nucleon, $\Lambda \approx n_\nu \langle \sigma v \rangle \sim G_{\rm F}^2 T^5$, where we have used $n_\nu \sim T^3$ and $\langle \sigma v \rangle \sim G_{\rm F}^2 T^2$ (see discussion following (3.20)). This yields

$$T_{\rm fr} \sim \left(\frac{g_\rho^{1/2}}{G_{\rm F}^2 M_{\rm P}} \right)^{1/3} \sim 1 \ {\rm MeV}, \tag{3.12}$$

i.e. freeze-out occurs at $t_{\rm fr} \approx 1\,{\rm sec}$ (using (2.66)). The neutron abundance at this time can be approximated by its equilibrium value (3.8),

$$X_{\rm n}(T_{\rm fr}) \simeq X_{\rm n}^{\rm eq}(T_{\rm fr}) = \left[1 + {\rm e}^{\Delta m/T_{\rm fr}} \right]^{-1} . \tag{3.13}$$

Since the exponent $\Delta m/T_{\rm fr}$ is of O(1), a substantial fraction of neutrons survive when chemical equilibrium between neutrons and protons is broken. This results, in turn, in the synthesis of a significant amount of helium in the early universe. It is interesting that the individual terms in the exponent above reflect the widest possible variety of physical interactions which apparently "conspire" to make this possible.† Also, the dependence of $T_{\rm fr}$ on the energy density driving the expansion makes the helium abundance sensitive to the number of relativistic particle species (e.g. massless neutrinos) present, or to any hypothetical non-relativistic particle which contributes appreciably to the energy density at this epoch.

Calculation of the asymptotically surviving abundance $X_{\rm n}(t \to \infty)$ requires explicit computation of the reaction rates (see Weinberg 1972)

$$\begin{aligned}
\lambda({\rm n}\nu_{\rm e} \to {\rm p}{\rm e}^-) &= A \int_0^\infty {\rm d}q_\nu \; q_\nu^2 \; q_{\rm e} \; E_{\rm e} \; (1 - f_{\rm e}) \; f_\nu \; , \qquad & E_{\rm e} &= E_\nu + \Delta m \; , \\
\lambda(n{\rm e}^+ \to {\rm p}\bar{\nu}_{\rm e}) &= A \int_0^\infty {\rm d}q_{\rm e} \; q_{\rm e}^2 \; q_\nu \; E_\nu \; (1 - f_\nu) \; f_{\rm e} \; , \qquad & E_\nu &= E_{\rm e} + \Delta m \; , \\
\lambda({\rm n} \to {\rm p}{\rm e}^-\bar{\nu}_{\rm e}) &= A \int_0^{q_0} {\rm d}q_{\rm e} \; q_{\rm e}^2 \; q_\nu \; E_\nu \; (1 - f_\nu)(1 - f_{\rm e}) \; , \quad & q_0 &= \sqrt{(\Delta m)^2 - m_{\rm e}^2} \; .
\end{aligned} \tag{3.14}$$

Here A is an effective coupling while $f_{\rm e}$ and f_ν are the distribution functions for electrons and neutrinos. Although the weak interaction coupling $G_{\rm F}$ is known quite accurately from muon decay, the value of A, or equivalently, the neutron lifetime, cannot be directly determined from this alone because neutrons and protons also interact strongly, hence

† The neutron-proton mass difference is determined by the strong and electromagnetic interactions, while the freeze-out temperature is fixed by the weak and gravitational interactions.

the ratio of the nucleonic axial vector (G_A) and vector (G_V) couplings is altered from unity (see Freedman 1990). Moreover, relating these couplings to the corresponding experimentally measured couplings for the u and d quarks is complicated by weak isospin violating effects. If we assume conservation of the weak vector current (CVC), then $G_\mathrm{V} = G_\mathrm{F}\cos\theta_\mathrm{C}$ where $\theta_\mathrm{C} \simeq 13^0$ is the Cabibbo angle which describes the mixing of the quark weak eigenstates into the mass eigenstates (see Marciano 1991). However the weak axial current is *not* conserved and G_A for nucleons differs from that for the first generation quarks. These non-perturbative effects cannot be reliably calculated, hence G_A (in practice, $G_\mathrm{A}/G_\mathrm{V}$) must be measured experimentally. The neutron lifetime is then given by

$$\tau_\mathrm{n}^{-1} = \frac{m_\mathrm{e}^5}{2\pi^3} G_\mathrm{V}^2 \left(1 + 3\,\frac{G_\mathrm{A}^2}{G_\mathrm{V}^2}\right)\ f\ , \qquad (3.15)$$

where $f = 1.715$ is the integral over the final state phase space (including Columb corrections) and G_V is usually determined directly from superallowed $0^+ \to 0^+$ pure Fermi decays of suitable light nuclei (see Wilkinson 1982). It is thus more reliable to measure the neutron lifetime directly if possible, and then relate it to the coupling A in (3.14) in order to obtain the other reaction rates.

Bernstein *et al* (1989) note that the major contribution to the integrals in (3.14) comes from particles of energy *higher* than the temperature during the BBN era, hence the Fermi-Dirac distributions may be approximated by their Boltzmann equivalents:

$$f_\mathrm{e} = \left[1 + \mathrm{e}^{E_\mathrm{e}/T_\mathrm{e}}\right]^{-1} \simeq \mathrm{e}^{-E_\mathrm{e}/T_\mathrm{e}}\ , \qquad f_\nu = \left[1 + \mathrm{e}^{E_\nu/t_\nu}\right]^{-1} \simeq \mathrm{e}^{-E_\nu/T_\nu}\ . \quad (3.16)$$

Also, since the Boltzmann weights are small in this dilute gas limit, the Pauli blocking factors in the reaction rates may be neglected:

$$1 - f_{e,\nu} \simeq 1\ , \qquad (3.17)$$

The electron temperature T_e above equals the photon temperature T but has been distinguished from the neutrino temperature T_ν because, as discussed in section 2.2, the annihilation of $\mathrm{e}^+\mathrm{e}^-$ pairs at $T \lesssim m_\mathrm{e}$ heats the photons and the (electromagnetically coupled) electrons but not the neutrinos which have become essentially non-interacting by this time. The evolution of T_ν/T is given by entropy conservation (2.70); numerical evaluation of this expression shows that T_ν remains within $\approx 10\%$ of T until $\approx 0.2\,\mathrm{MeV}$, by which time, as we shall see below, neutron freeze-out is effectively over. Hence Bernstein *et al* (1989) assume that $T_\nu = T$; the detailed balance condition (3.5) follows from comparison of the rates (3.14) to the corresponding rates for the reverse processes. Their final approximation is to set $m_\mathrm{e} = 0$ in evaluating $\lambda\,(\mathrm{n}\nu_\mathrm{e} \to \mathrm{p}\mathrm{e}^-)$ and $\lambda\,(\mathrm{n}\mathrm{e}^+ \to \mathrm{p}\bar{\nu}_\mathrm{e})$ which get most of their contribution from energies $E_{\mathrm{e},\nu} \gg m_\mathrm{e}$. These rates are then equal and given by the formula

$$\lambda\ (\mathrm{n}\nu_\mathrm{e} \to \mathrm{p}\mathrm{e}^-) = \lambda(\mathrm{n}\mathrm{e}^+ \to \mathrm{p}\bar{\nu}_\mathrm{e}) = A\ T^3\ [24\ T^2 + 12\ T\ \Delta m + 2\ (\Delta m)^2]\ , \qquad (3.18)$$

which is accurate to better than 15% until T drops to m_{e} by which time the rates themselves have become very small. Integration of the neutron beta decay rate (see (3.14)) now gives the desired relation between the coupling A and the neutron lifetime τ_{n}:

$$\begin{aligned}\frac{1}{\tau_{\mathrm{n}}} = \frac{A}{5}\sqrt{(\Delta m)^2 - m_{\mathrm{e}}^2}\left[\frac{1}{6}(\Delta m)^4 - \frac{3}{4}(\Delta m)^2 m_{\mathrm{e}}^2 - \frac{2}{3}m_{\mathrm{e}}^4\right] + \frac{A}{4}m_{\mathrm{e}}^4 \Delta m \cosh^{-1}\left(\frac{\Delta m}{m_{\mathrm{e}}}\right) \\ = 0.0158\ A\ (\Delta m)^5\ .\end{aligned} \quad (3.19)$$

Hence the total reaction rate can be expressed in terms of the neutron lifetime as,

$$\begin{aligned}&\lambda_{\mathrm{np}}(t) \simeq 2\ \lambda(\mathrm{n}\nu_{\mathrm{e}} \to \mathrm{pe}^-) = \frac{a}{\tau_{\mathrm{n}} y^5}(12 + 6y + y^2), \\ &y \equiv \frac{\Delta m}{T}, \qquad a = 253.\end{aligned} \quad (3.20)$$

The contribution of neutron decay itself to λ_{np} has been neglected here since it is unimportant during the freeze-out period and becomes comparable to the other terms only for $T \leq 0.13$ MeV (corresponding to $y > 10$). We see that for $T \gg \Delta m$, i.e. $y \ll 1$, the reaction rate is $\lambda \approx 12a/\tau_{\mathrm{n}} y^5$, which we have approximated earlier as $\lambda \sim G_{\mathrm{F}}^2 T^5$ (using (3.15)) in order to estimate T_{fr} (3.12).

The integrating factor in (3.10) can now be calculated:

$$\begin{aligned}I(y, y') &= \exp\left[-\int_{y'}^{y} \mathrm{d}y'' \frac{\mathrm{d}t''}{\mathrm{d}y''}\ \Lambda(y'')\right] \\ &= \exp\left[K(y) - K(y')\right]\ ,\end{aligned} \quad (3.21)$$

where,

$$\begin{aligned}K(y) \quad &\equiv -b\int_{\infty}^{y} \mathrm{d}y' \left[\frac{12}{y'^4} + \frac{6}{y'^3} + \frac{1}{y'^2}\right](1 + \mathrm{e}^{-y'}) \\ &= b\left[\left(\frac{4}{y^3} + \frac{3}{y^2} + \frac{1}{y}\right) + \left(\frac{4}{y^3} + \frac{1}{y^2}\right)\mathrm{e}^{-y}\right], \\ \text{and,} \quad b &= a\left(\frac{45}{4\pi^3 g_\rho}\right)^{1/2} \frac{M_{\mathrm{P}}}{\tau_{\mathrm{n}}\,(\Delta m)^2}.\end{aligned} \quad (3.22)$$

The neutron abundance is therefore

$$X_{\mathrm{n}}(y) = X_{\mathrm{n}}^{\mathrm{eq}}(y) + \int_0^y \mathrm{d}y'\ \mathrm{e}^{y'}[X_{\mathrm{n}}^{\mathrm{eq}}(y')]^2\ \exp[K(y) - K(y')]. \quad (3.23)$$

The integral can be easily evaluated numerically once the value of b is specified. In the Standard Model, the number of relativistic degrees of freedom corresponding to photons, electrons and positrons and 3 species of massless † neutrinos ($N_\nu = 3$) is $g_\rho = 43/4$ at

† The effects on BBN of a finite neutrino mass are discussed in section 5.1. The results of standard BBN are however unaffected for $m_\nu \lesssim 0.1\,\mathrm{MeV}$ (Kolb and Scherrer 1982). Experimentally, it is only

this time, hence $b = 0.252$, taking $\tau_{\rm n} = 887\,{\rm sec}$ (3.48). (Subsequently g_ρ drops to 3.36 following e^+e^- annihilation (2.71); this raises the total energy density in relativistic particles but the error incurred by ignoring this is negligible since $X_{\rm n}$ has essentially stopped evolving by then; also Bernstein *et al* (1989) actually used $\tau_{\rm n} = 896\,{\rm sec}$ but we have corrected their numbers.) This yields the asymptotic abundance

$$X_{\rm n}(y \to \infty) = 0.150 \; , \tag{3.24}$$

which is already achieved by the time T has dropped to about $0.25\,{\rm MeV}$ ($y \simeq 5$), corresponding to $t \simeq 20\,{\rm sec}$.

3.1.2. Element synthesis: Having dealt with the breaking of weak equilibrium between neutrons and protons, we now consider the onset of nuclear reactions which build up the light nuclei. This has been traditionally studied by numerical solution of the complete nuclear reaction network (Peebles 1966b, Wagoner *et al* 1967, Wagoner 1969, 1973). More recently the coupled balance equations for the elemental abundances have been semi-analytically solved by a novel method of fixed points (Esmailzadeh *et al* 1991) as discussed later. First we outline the essential physical processes as they pertain to the calculation of the ^{4}He abundance.

Neutrons and protons react with each other to build up light nuclei through the following sequence of two-body reactions:

$$\begin{aligned}
&{\rm p}({\rm n},\gamma){\rm D}, \\
&{\rm D}({\rm p},\gamma)^3{\rm He}, \quad {\rm D}({\rm D},{\rm n})^3{\rm He}, \quad {\rm D}({\rm D},{\rm p}){\rm T}, \\
&{\rm T}({\rm D},{\rm n})^4{\rm He}, \quad {\rm T}(^4{\rm He},\gamma)^7{\rm Li}, \\
&^3{\rm He}({\rm n},{\rm p}){\rm T}, \quad ^3{\rm He}({\rm D},{\rm p})^4{\rm He}, \quad ^3{\rm He}(^4{\rm He},\gamma)^7{\rm Be}, \\
&^7{\rm Li}({\rm p},^4{\rm He})^4{\rm He}, \quad ^7{\rm Be}({\rm n},{\rm p})^7{\rm Li} \\
&\vdots
\end{aligned} \tag{3.25}$$

The first reaction is the most crucial since deuterium must be formed in appreciable quantity before the other reactions can proceed at all, the number densities being in general too low to allow nuclei to be built up directly by many-body reactions such as $2{\rm n} + 2{\rm p} \to {}^4{\rm He}$. The rate (per neutron) of this reaction (see Weinberg 1972),

$$\lambda({\rm np} \to {\rm D}\gamma) = 4.55 \times 10^{-20} \; n_{\rm p} \; {\rm cm}^3{\rm sec}^{-1}, \tag{3.26}$$

is quite large, being determined by the strong interactions, and exceeds the expansion rate down to quite low temperatures of ${\rm O}(10^{-3})\,{\rm MeV}$. Hence at the epoch of interest,

known that $m_{\nu_{\rm e}} < 5.1\,{\rm eV}$, $m_{\nu_\mu} < 160\,{\rm keV}$, and $m_{\nu_\tau} < 24\,{\rm MeV}$ (Particle Data Group 1996). Hence the $\nu_{\rm e}$, and probably the ν_μ too, are indeed effectively massless but the ν_τ can, in priniple, play a more complex role in BBN.

deuterium will indeed be present with its equilibrium abundance, given by the Saha equation

$$\frac{n_{\rm D}}{n_{\rm n}n_{\rm p}} = \frac{g_{\rm D}}{g_{\rm n}g_{\rm p}}\left(\frac{m_{\rm D}}{m_{\rm n}m_{\rm p}}\right)^{3/2}\left(\frac{T}{2\pi}\right)^{-3/2}{\rm e}^{\Delta_{\rm D}/T} , \tag{3.27}$$

where $\Delta_{\rm D}\equiv m_{\rm n}+m_{\rm p}-m_{\rm D}=2.23\,{\rm MeV}$ is the deuteron binding energy, and the g's are statistical factors. This can be rewritten in terms of the respective mass fractions as

$$\frac{X_{\rm D}}{X_{\rm n}X_{\rm p}} \simeq \frac{24\zeta(3)}{\sqrt{\pi}}\eta\left[\frac{T}{m_{\rm p}}\right]^{3/2}{\rm e}^{\Delta_{\rm D}/T}, \quad X_i \equiv \frac{n_i A_i}{n_{\rm N}} , \tag{3.28}$$

where,

$$\eta \equiv \frac{n_{\rm N}}{n_\gamma} = 2.722\times10^{-8}\Omega_{\rm N}h^2\Theta^{-3} , \tag{3.29}$$

is the ratio of the total number of nucleons (bound or free) to the number of photons (which remains constant following e^+e^- annihilation). This quantity is not well known observationally because it is not clear how much of the dark matter in the universe is in the form of nucleons. An audit of luminous material in galaxies and X-ray emitting gas in clusters provides the lower limit (Persic and Salucci 1992):

$$\Omega_{\rm N} \equiv \frac{\rho_{\rm N}}{\rho_{\rm c}} > 2.2\times10^{-3} + 6.1\times10^{-4}h^{-1.3} . \tag{3.30}$$

(Henceforth, we omit the subscript $_0$ on Ω and $\Omega_{\rm N}$.) A conservative upper limit follows from assuming that all the gravitating matter permitted by the present age and expansion rate of the universe is made up of nucleons, i.e. $\Omega_{\rm N}h^2 \lesssim 1$ (2.28). (Such a high density purely nucleonic universe cannot create the observed large-scale structure, given primordial 'adiabatic' density fluctuations; however a viable model *can* be constructed assuming primordial isocurvature fluctuations (Peebles 1987, Cen *et al* 1993) which satisfies CMB anisotropy constraints with $\Omega_{\rm N} \lesssim 1$ (e.g. Sugiyama and Silk 1994).) These considerations require the value of η today to lie in the rather broad range:

$$1.8\times10^{-11} \lesssim \eta \lesssim 2.8\times10^{-8}, \tag{3.31}$$

using the limits $0.4 \lesssim h \lesssim 1$ (2.17) and $0.993 < \Theta < 1.007$ (2.23). In the Standard Model, these constraints also apply during nucleosynthesis since e^+e^- annihilation is effectively over by this epoch so the comoving photon number, hence η, does not change further.

If deuterium synthesis is assumed to begin at a temperature $T_{\rm ns}$ when $X_{\rm D}/X_{\rm n}X_{\rm p}$ becomes of O(1), then for a typical value $\eta = 5\times10^{-10}$, (3.28) gives $T_{\rm ns} \simeq \Delta_{\rm D}/34$, an estimate which is only logarithmically sensitive to the adopted nucleon density.†

† Naïvely we would expect deuterium synthesis to begin as soon as the average blackbody photon

Bernstein *et al* (1989) obtain a more careful estimate by examination of the rate equation governing the deuterium abundance. Defining the onset of nucleosynthesis by the criterion $\mathrm{d}X_{\mathrm{D}}/\mathrm{d}z = 0$ at $z = z_{\mathrm{ns}}$ (where $z \equiv \Delta_{\mathrm{D}}/T$), they find that the critical temperature is given by the condition

$$2.9 \times 10^{-6} \left(\frac{\eta}{5 \times 10^{-10}}\right)^2 z_{\mathrm{ns}}^{-17/6} \exp(-1.44 z_{\mathrm{ns}}^{1/3})\, \mathrm{e}^{z_{\mathrm{ns}}} \simeq 1. \tag{3.32}$$

Taking $\eta = 5 \times 10^{-10}$, this gives $z_{\mathrm{ns}} \simeq 26$, i.e.

$$T_{\mathrm{ns}} \simeq \frac{\Delta_{\mathrm{D}}}{26} = 0.086\,\mathrm{MeV}\,. \tag{3.33}$$

At this epoch, $g_\rho = 3.36$ (2.71), hence the time-temperature relationship (2.66) says that nucleosynthesis begins at

$$t_{\mathrm{ns}} \simeq 180\,\mathrm{sec}, \tag{3.34}$$

as widely popularized by Weinberg (1977).

By this time the neutron abundance surviving at freeze-out has been depleted by β-decay to

$$X_{\mathrm{n}}(t_{\mathrm{ns}}) \simeq X_{\mathrm{n}}(y \to \infty)\, \mathrm{e}^{-t_{\mathrm{ns}}/\tau_{\mathrm{n}}} = 0.122\ . \tag{3.35}$$

Nearly *all* of these surviving neutrons are captured in $^4\mathrm{He}$ because of its large binding energy ($\Delta_{^4\mathrm{He}} = 28.3\,\mathrm{MeV}$) via the reactions listed in (3.25). Heavier nuclei do not form in any significant quantity both because of the absence of stable nuclei with A=5 or 8 which impedes nucleosynthesis via $\mathrm{n}\,^4\mathrm{He}$, $\mathrm{p}\,^4\mathrm{He}$ or $^4\mathrm{He}\,^4\mathrm{He}$ reactions, and the large Coulomb barrier for reactions such as $\mathrm{T}(^4\mathrm{He},\gamma)^7\mathrm{Li}$ and $^3\mathrm{He}(^4\mathrm{He},\gamma)^7\mathrm{Be}$.† Hence the resulting *mass* fraction of helium, conventionally referred to as $Y_{\mathrm{p}}(^4\mathrm{He})$, is simply given by

$$Y_{\mathrm{p}}(^4\mathrm{He}) \simeq 2\, X_{\mathrm{n}}(t_{\mathrm{ns}}) = 0.245\ , \tag{3.36}$$

where the subscript p denotes primordial. The above calculation makes transparent how the synthesized helium abundance depends on the physical parameters. The dominant

energy of $2.7T$ falls below Δ_{D} since deuterons would then presumably no longer be photodissociated as soon as they are formed. However, since there are $\sim 10^{10}$ photons per nucleon, there are still enough high energy photons in the Wien tail of the Planck distribution at this time which are capable of photodissociating deuterons, and it takes rather longer for the 'deuterium bottleneck' to break. There is, in fact, another contributory reason, which we will discuss following (3.47).

† If there are large fluctuations in the nucleon density, such as may be induced by a first-order quark-hadron phase transition (see Reeves 1991), then differential transport of neutrons and protons creates neutron-rich regions where heavy elements can indeed be formed through reactions such as $\mathrm{H}(\mathrm{n},\gamma)\,\mathrm{D}(\mathrm{n},\gamma)\,\mathrm{T}(\mathrm{D},\mathrm{n})^4\mathrm{He}(\mathrm{T},\gamma)^7\mathrm{Li}(\mathrm{n},\gamma)^8\mathrm{Li}(^4\mathrm{He},\mathrm{n})^{11}\mathrm{B}(\mathrm{n},\gamma)^{12}\mathrm{B}(\mathrm{e},\nu_{\mathrm{e}})^{12}\mathrm{C}(\mathrm{n},\gamma)^{13}\mathrm{C}(\mathrm{n},\gamma)^{14}\mathrm{C}\ldots$ (see Malaney and Mathews 1993). This will be discussed further in section 3.3.

effect of a smaller neutron lifetime $\tau_{\rm n}$ is that freeze-out occurs at a lower temperature with a smaller neutron fraction ((3.22) and (3.23)), hence less ^{4}He is subsequently synthesized; this is only partly negated by the larger β-decay factor (3.35) since only $\approx$ 20% of the neutrons have decayed when nucleosynthesis begins. Increasing the assumed number of relativistic neutrino species N_ν increases g_ρ (2.71), speeding up the expansion and leading to earlier freeze-out and earlier onset of nucleosynthesis, hence a larger helium abundance. Finally as the nucleon-to-photon ratio η increases, the 'deuterium bottleneck' is broken increasingly earlier (see (3.28)), allowing a larger fraction of neutrons to survive β-decay and be burnt to ^{4}He, the abundance of which thus rises approximately logarithmically with η.

Bernstein *et al* (1989) also consider the effect on neutron freeze-out of a possible excess of electron neutrinos over antineutrinos, parametrized by a dimensionless chemical potential, $\xi_{\nu_{\rm e}} \equiv \mu_{\nu_{\rm e}}/T$, which remains constant for freely expanding neutrinos (see (2.31)). Anticipating that $\xi_{\nu_{\rm e}}$ will be constrained to be sufficiently small, they neglect the slight increase in expansion rate due to the increased energy density of the neutrinos and consider only the effect on neutron-proton interconversions. (They do not consider a chemical potential for other neutrino types, which would only add to the energy density without affecting the weak reactions.) The resultant increase in the rate of $n\nu_{\rm e}\rightarrow p{\rm e}^-$ alters the detailed balance equation (3.5) to

$$\lambda_{\rm pn} = \lambda_{\rm np}\ \exp\left[-\frac{\Delta m}{T(t)} - \xi_{\nu_{\rm e}}\right], \qquad \xi_{\nu_{\rm e}} \equiv \frac{\mu_{\nu_{\rm e}}}{T}\ , \tag{3.37}$$

and, hence, lowers the equilibrium neutron abundance to,

$$X_{\rm n}^{\rm eq}(t,\xi_{\nu_{\rm e}}) = \left[1 + {\rm e}^{(y+\xi_{\nu_{\rm e}})}\right]^{-1}, \qquad y \equiv \frac{\Delta m}{T(t)}\ . \tag{3.38}$$

Bernstein *et al* (1989) find that this alters the asymptotic neutron abundance by the *same* factor, viz.

$$X_{\rm n}(\xi_{\nu_{\rm e}}, y\rightarrow\infty) = {\rm e}^{-\xi_{\nu_{\rm e}}} X_{\rm n}(y\rightarrow\infty)\ . \tag{3.39}$$

It is now easily shown that the synthesized helium mass fraction depends on the relevant parameters as

$$Y_{\rm p}(^4{\rm He}) = 0.245 + 0.014\,\Delta N_\nu + 0.0002\,\Delta\tau_{\rm n} + 0.009\ \ln\left(\frac{\eta}{5\times10^{-10}}\right) - 0.25\,\xi_{\nu_{\rm e}}\ , \tag{3.40}$$
$$\text{where,} \qquad \Delta N_\nu \equiv N_\nu - 3\ , \qquad \Delta\tau_{\rm n} \equiv \tau_{\rm n} - 887\,{\rm sec}\ .$$

For comparison, a recent numerical solution (Walker *et al* 1991) of the nuclear reaction network finds that the helium yield is fitted (to within ±0.001) in the nucleon density range $3\times10^{-10} \lesssim \eta \lesssim 10^{-9}$ by the formula

$$Y_{\rm p}(^4{\rm He}) = 0.244 + 0.012\ \Delta N_\nu + 0.00021\,\Delta\tau_{\rm n} + 0.01\ \ln\left(\frac{\eta}{5\times10^{-10}}\right)\ . \tag{3.41}$$

(There is no term here corresponding to neutrino degeneracy since the effect of this has not been parametrized by numerical means.) We see that the semi-analytic result of Bernstein *et al* (1989) is impressively accurate.

Small amounts of deuterium ($X_{\rm D} \sim 10^{-4}$), helium-3 ($X_{^3{\rm He}} \sim 10^{-5}$) and lithium-7 ($X_{^7{\rm Li}} \sim 10^{-9}$) are also left behind when the nuclear reaction rates fall behind the expansion rate and BBN ends, at $t_{\rm end} \approx 1000\,{\rm sec}$. (The helium-3 abundance is taken to include that of surviving tritium which subsequently undergoes beta decay and, similarly, the lithium-7 abundance includes that of beryllium-7.) In contrast to ^{4}He, the abundances of these elements are quite sensitive to the nucleon density since this directly determines the two-body nuclear reaction rates. The D and ^{3}He abundances drop rapidly with increasing η which ensures more efficient burning to ^{4}He. The ^{7}Li abundance also decreases with increasing η in a regime where its abundance is determined by the competition between $^4{\rm He}({\rm T},\gamma)^7{\rm Li}$ and $^7{\rm Li}({\rm p},{}^4{\rm He})^4{\rm He}$; however at sufficiently high η ($\gtrsim 3\times10^{-10}$), its abundance begins increasing again with η due to the increasing production of ^{7}Be through $^4{\rm He}(^3{\rm He},\gamma)^7{\rm Be}$, which makes ^{7}Li by electron capture, $^7{\rm Be}({\rm e}^-,\nu_{\rm e})^7{\rm Li}$. The reaction rates for the synthesis of $A > 7$ nuclei are not all well known but, even with extreme values chosen, the mass fraction of elements such as Be and B does not exceed 10^{-13} for any value of η (e.g. Thomas *et al* 1993).

These results concerning the abundances of D, ^{3}He and ^{7}Li were originally obtained by numerical solution of the complete nuclear reaction network (e.g. Wagoner 1969). More recently, Esmailzadeh *et al* (1991) have shown that these abundances are given to good accuracy by the fixed points of the corresponding rate equations, as discussed below. The general equation governing the abundance of a given element is

$$\frac{{\rm d}X}{{\rm d}t} = J(t) - \Gamma(t)X \ , \tag{3.42}$$

where $J(t)$ and $\Gamma(t)$ are the time-dependent source and sink terms, which, in general, depend on the abundances of the other elements. The solution to this equation is (Dimopoulos *et al* 1988)

$$X(t) = \exp\left(-\int_{t_{\rm i}}^{t}{\rm d}t'\ \Gamma(t')\right)\left[X(t_{\rm i}) + \int_{t_{\rm i}}^{t}{\rm d}t'\ J(t')\ \exp\left(-\int_{t_{\rm i}}^{t}{\rm d}t''\ \Gamma(t'')\right)\right] \ , \tag{3.43}$$

where $t_{\rm i}$, the initial time, may be taken to be zero. These authors show that if

$$\left|\frac{\dot{J}}{J} - \frac{\dot{\Gamma}}{\Gamma}\right| \ll \Gamma \ , \tag{3.44}$$

then X approaches its equilibrium value †

$$X^{\rm eq} = \frac{J(t)}{\Gamma(t)} \tag{3.45}$$

on a time scale of $\mathrm{O}(\Gamma^{-1})$. This state is dubbed 'quasi-static equilibrium' (QSE) since the source and sink terms nearly cancel each other such that $\dot{X} \simeq 0$. (Note that since $\dot{\Gamma}/\Gamma \approx \dot{J}/J \approx H$, the condition (3.44) is somewhat more stringent than $\Gamma \gg H$ which would be the naïve criterion for QSE.) As the universe expands, the nuclear reaction rates slow down rapidly due to the dilution of particle densities and the increasing importance of Coulomb barriers; hence J and Γ fall rapidly with time. At some stage $t = t_{\rm fr}$, X can be said to 'freeze-out' if its value does not change appreciably beyond that point, i.e. if

$$\int_{t_{\rm fr}}^{\infty} {\rm d}t\ J(t) \ll X(t_{\rm fr}) \ , \qquad \int_{t_{\rm fr}}^{\infty} {\rm d}t\ \Gamma(t) \ll 1 \ . \tag{3.46}$$

Generally freeze-out occurs when $\Gamma \simeq H$ and the asymptotic value of the elemental abundance is then given by

$$X(t \to \infty) \simeq X^{\rm eq}(t_{\rm fr}) = \frac{J(t_{\rm fr})}{\Gamma(t_{\rm fr})} \ . \tag{3.47}$$

It now remains to identify the largest source and sink terms for each element and calculate the freeze-out temperature and the QSE abundance at this epoch. This requires careful examination of the reaction network and details of this procedure are given by Esmailzadeh *et al* (1991). These authors study the time development of the abundances for a particular choice of the nucleon density and also calculate the final abundances as a function of the nucleon density. As shown in figure 3, the agreement between their analytic approximations (dotted lines) and the exact numerical solutions (full lines) is impressive. The abundances of D, ^{3}He and ^{7}Li are predicted correctly to within a factor of ≈ 3 for the entire range $\Omega_{\rm N} \sim 0.001 - 1$, and even the abundance of ^{4}He is obtained to better than 5% for $\Omega_{\rm N} \sim 0.01 - 1$. Moreover, this analysis clarifies several features of the underlying physics. For example, it becomes clear that in addition to the 'deuterium bottleneck' alluded to earlier (see footnote concerning (3.32)), the synthesis of ^{4}He is additionally delayed until enough tritium has been synthesized through ${\rm D(D},p){\rm T}$, since the main process for making ^{4}He is ${\rm D(T},n)^4{\rm He}$. In fact, D and T are both in QSE when ^{4}He forms, hence the former reaction is the *only* one whose cross-section has any perceptible influence on the ^{4}He abundance. This behaviour is illustrated in figure 3 where the abundance of ^{4}He is seen to depart from its NSE curve

† This is distinct from the value in nuclear statistical equilibrium (NSE) which is given by the Saha equation (3.28) and increases exponentially as the temperature drops, as shown by the dashed lines in figure 3. Considerations of NSE alone are not useful in the present context where the Hubble expansion introduces a time-scale into the problem (cf. Kolb and Turner 1990).

(dashed line) at about 0.6 MeV and follow the abundances of T and ^{3}He until these too depart from NSE at about 0.2 MeV due to the 'deuterium bottleneck'; subsequently ^{4}He, ^{3}He and T all follow the evolution of D until it finally deviates from NSE at about 0.07 MeV (see Smith *et al* 1993).

In figures 3, 4 and 5 we show the elemental yields in the standard Big Bang cosmology (with $N_\nu = 3$) obtained using the Wagoner (1969, 1973) computer code, which has been significantly improved and updated by Kawano (1988, 1992),† incorporating both new measurements and revised estimates of the nuclear cross-sections (Fowler *et al* 1975, Harris *et al* 1983, Caughlan *et al* 1985, Caughlan and Fowler 1988). Figure 3 shows the evolution of the abundances (by number) with decreasing temperature for a specific choice of the nucleon density ($\Omega_{\rm N}h^2 = 0.01 \Rightarrow \eta = 2.81 \times 10^{-10}$) while figure 4 shows the dependence of the final abundances on η. In figure 5 we show these in more detail, along with their associated uncertainties calculated by Krauss and Kernan (1995) as discussed below. This last figure displays the mass fraction $Y_{\rm p}(^4{\rm He})$ on a *linear* scale for clarity.

3.1.3. Uncertainties: There have been many studies of the theoretical uncertainties in the predicted abundances (e.g. Beaudet and Reeves 1983, Yang *et al* 1984, Delbourgo-Salvador *et al* 1985, Kajino *et al* 1987, Riley and Irvine 1991), in particular that of ^{7}Li (Kawano *et al* 1988, Deliyannis *et al* 1989). Because of the complex interplay between different nuclear reactions, it is not straightforward to assess the effect on a particular elemental yield of the uncertainty in some reaction rate. An illuminating Monte Carlo analysis by Krauss and Romanelli (1990) exhibited the effect on the abundances corresponding to *simultaneous* variations in all relevant nuclear reaction rates by sampling them from Gaussian distributions centred on the appropriate mean values and with widths corresponding to the experimental uncertainties. This exercise was redone by Smith *et al* (1993) using the latest cross-sections for the eleven most important nuclear reactions (3.25) and their estimated uncertainties (which are temperature dependent for $^3{\rm He}(^4{\rm He},\gamma)^7{\rm Be}$ and ${\rm T}(^4{\rm He},\gamma)^7{\rm Li}$). These authors carefully discussed the statistical and systematic uncertainties in the laboratory measurements of relevant cross-sections and emphasized, in particular, the uncertainties in the 'S-factor' which enters in the extrapolation of a measured cross-section down in energy to obtain its thermally averaged value at temperatures relevant to nucleosynthesis. Recently Krauss and Kernan (1995) (see also Kernan and Krauss 1994) have repeated the exercise with an improved Monte Carlo procedure, the latest value for $\tau_{\rm n}$ (3.48) and the new cross-section for the secondary reaction $^7{\rm Be}({\rm p},\gamma)^8{\rm B}$ (which however does not affect the results perceptibly). The dashed lines in figure 5 indicate the region within

† This code has been made publicly available by L Kawano and has become the *de facto* standard tool for BBN studies, enabling easy comparison of results obtained by different researchers.

which 95% of the computed values fall, which thus correspond to "2σ" bounds on the predicted abundances.

The major uncertainty in the ^{4}He abundance is due to the experimental uncertainty in the neutron lifetime. For many years there were large discrepancies between different measurements of $\tau_{\rm n}$ suggestive of unknown systematic errors (see Byrne 1982). Until recently most BBN calculations adopted a relatively high value, viz. *upwards* of 900 sec (Yang *et al* 1984, Boesgaard and Steigman 1985, Steigman *et al* 1986), although Ellis *et al* (1986b) cautioned that a significantly lower value of 898 ± 6 sec was indicated (using (3.15)) by the precision measurement of $G_{\rm A}/G_{\rm V} = -1.262 \pm 0.004$ obtained using polarized neutrons (Bopp *et al* 1986). Subsequently several precise direct measurements using 'bottled' neutrons (see Dubbers 1991, Schreckenbach and Mampe 1992) have shown that the lifetime is indeed lower than was previously believed. The present weighted average is (Particle Data Group 1994, 1996) †

$$\tau_{\rm n} = 887 \pm 2 \ \text{sec} , \tag{3.48}$$

as used in our computations and in other recent work (e.g. Krauss and Kernan 1995, Hata *et al* 1995, Olive and Scully 1996, Kernan and Sarkar 1996). As we will see in section 4.1 the *lower* bound is of particular importance in using BBN to set constraints on new particles. Variation of the neutron lifetime by 2σ causes $Y_{\rm p}(^4{\rm He})$ to change by less than 0.4%, while the effect on the other elemental abundances is comparable, hence negligible in comparison to their other uncertainties. The uncertainties in the nuclear cross-sections can alter the calculated yields of D and ^{3}He by upto $\approx 15\%$ and ^{7}Li by upto $\approx 50\%$ but have little effect ($\lesssim 0.5\%$) on the ^{4}He abundance. As mentioned earlier, the effect of these uncertainties on the final abundances are correlated, hence best studied by Monte Carlo methods.

Finally, there are computational errors associated with the integration routine in the numerical code for BBN, which can be upto a few per cent for D, ^{3}He and ^{7}Li but $\lesssim 0.1\%$ for ^{4}He, with the default settings of the time steps (Kawano 1992). These have been compensated for by Smith *et al* (1993) but not always taken into account in earlier work. Kernan (1993) has explored this question in more detail and states that making the integration time steps short enough that different (order) Runge-Kutta drivers converge on the same result can produce an increase in $Y_{\rm p}$ of as much as $+0.0017$ relative to results obtained with the default step size. However direct comparison between the results of Kernan and Krauss (1994) and those of Walker *et al* (1991) and Smith *et al*

† The Particle Data Group (1990) had previously quoted an weighted average $\tau_{\rm n} = 888.6 \pm 3.5$ sec and Walker *et al* (1991) adopted the 95% c.l. range 882-896 sec. Smith *et al* (1993) considered only post-1986 experiments which give $\tau_{\rm n} = 888.5 \pm 1.9$ sec but doubled the uncertainty to ± 3.8 sec in their analysis. Other recent papers (e.g. Kernan and Krauss 1994, Copi *et al* 1995a) use the Particle Data Group (1992) value of 889.1 ± 2.1 sec.

(1993) does not reveal any difference due to this reason.

Given the recent sharp fall in the uncertainty in the neutron lifetime, it is important to include all corrections to the weak interaction rates which have a comparable effect on the ^{4}He abundance. The most detailed such study by Dicus *et al* (1982) (see also Cambier *et al* 1982, Baier *et al* 1990) finds that including Coulomb corrections to the weak interaction rates decreases the calculated value Y by 0.0009, while the corrections due to zero temperature radiative corrections ($\Delta Y = +0.0005$), finite temperature radiative corrections ($\Delta Y = -0.0004$), plasma effects on the electron mass ($\Delta Y = +0.0001$) and, finally, the slight heating of the neutrinos by e^+e^- annihilation ($\Delta Y = -0.0002$),† taken together, change Y by less than 0.0001. Dicus *et al* (1982) also state that Y decreases systematically by 0.0013 when the weak rates are computed by numerical integration rather than being obtained from the approximate fitting formula given by Wagoner (1973). This amounts to a total decrease in Y of 0.0022 for the parameter values ($\eta = 3 \times 10^{-10}, N_\nu = 3, \tau_\mathrm{n} = 918\,\mathrm{sec}$) they adopted; by varying these over a wide range ($\eta = 0.3 - 30 \times 10^{-10},\ N_\nu = 2 - 10,\ \tau_\mathrm{n} = 693 - 961\,\mathrm{sec}$) Dicus *et al* find an average systematic change of $\Delta Y = -0.0025$. Smith *et al* (1993) apply this correction to their results obtained using the fitted rates while Walker *et al* (1991) integrate the rates numerically with the Coulomb corrections and neutrino heating included, using a code updated to Caughlan and Fowler's (1988) cross-sections, and state that the residual uncertainty in Y_p due to all other effects does not exceed ± 0.0002. (The effect of all these corrections on the D, ^{3}He and ^{7}Li abundances is only $1 - 2\%$, hence negligible in comparison with the other uncertainties for these elements.)

Subsequently, Kernan (1993) has carefully reexamined the small corrections discussed by Dicus *et al* (1982); although his conclusions differ in detail, the net correction he finds for Coulomb, radiative and finite temperature effects is fortuitously the same, viz. $\Delta Y \simeq -0.009$. Further, Seckel (1993) has drawn attention to the effects of finite nucleon mass which cause a slight ($\approx 1\%$) decrease in the weak reaction rates. He finds that Y_p has been systematically *underestimated* by about 0.0012 in all previous work which ignored such effects.‡ The fractional changes in the abundances of the other light elements due to nucleon mass effects is $\lesssim 1\%$.

† Rana and Mitra (1991) have claimed that neutrino heating causes a large change, $\Delta Y \simeq -0.003$. However by incorporating a careful analysis of neutrino heating by Dodelson and Turner (1992) into the BBN code, Fields *et al* (1993) find $\Delta Y = +0.00015$ for $\eta \sim 10^{-10} - 10^{-9}$, comparable in magnitude to Dicus *et al* 's estimate although of *opposite* sign. Hannestad and Madsen (1995) obtain $\Delta Y = +0.0001$ from a similar analysis incorporating full Fermi-Dirac statistics.

‡ Gyuk and Turner (1993) have incorporated Seckel's calculations into the BBN code and state that the actual correction ranges between 0.0004 and 0.0015 over the range $\eta \sim 10^{-11} - 10^{-8}$, being well approximated by $+0.0057\,Y$. However we prefer to follow Seckel's original analysis which suggests that the correction is η-independent.

The abundances shown in figures 3-5 have been computed by explicit integration of the weak rates using Kawano's (1992) code, with the lowest possible settings of the time steps in the (2nd order) Runge-Kutta routine, which allows rapid convergence to within 0.01% of the true value (Kernan 1993). (We find that doing so increases Y by 0.0003 on average (for $\eta \sim 10^{-10} - 10^{-9}$) relative to the value obtained with the default settings. Also, explicitly integrating the weak rates reduces Y by 0.0009 on average relative to using fitting formulae.) To this calculated value Y we apply a net correction of +0.0003, obtained by adopting the Coulomb, radiative and finite temperature corrections recalculated by Kernan (1993) (which includes the new Fields *et al* (1993) estimate of neutrino heating) and the correction for finite nucleon mass given by Seckel (1993). All this results in an average *increase* of 0.0027 in $Y_{\rm p}$ relative to the values quoted by Smith *et al* (1993), i.e. the true value is fortuitously almost *identical* to that obtained by Wagoner's (1973) procedure of using fitted rates and a coarse integration mesh and ignoring all corrections!

3.1.4. Elemental yields: For comparison with the previously given formulae (3.40) and (3.41), our best fit over the range $\eta \sim 3 \times 10^{-10} - 10^{-9}$ is:

$$Y_{\rm p}(^4{\rm He}) = 0.2459 + 0.013\ \Delta\ N_\nu + 0.0002\ \Delta\tau_{\rm n} + 0.01\ \ln\left(\frac{\eta}{5\times10^{-10}}\right)\ . \qquad (3.49)$$

However, as is evident from Figure 5 , any log-linear fit of this kind overestimates $Y_{\rm p}$ for $\eta \lesssim 3 \times 10^{-10}$. A better fit (to within ±0.1%) over the entire range $\eta = 10^{-10} - 10^{-9}$ is given for the Standard Model ($N_\nu = 3$) by

$$Y_{\rm p}(^4{\rm He}) = 0.2462 + 0.01\ \ln\left(\frac{\eta}{5\times10^{-10}}\right)\left(\frac{\eta}{5\times10^{-10}}\right)^{-0.2} \pm 0.0012\ . \qquad (3.50)$$

We have indicated the typical 2σ error which results, in about equal parts, from the uncertainty in the neutron lifetime (3.48) and in the nuclear reaction rates. (As shown in figure 5, the error determined by Monte Carlo actually varies a bit with η.) Our values for $Y_{\rm p}$ are systematically higher by about 0.0005 than those shown in figure 5 as obtained by Krauss and Kernan (1995) who use the same neutron lifetime. Such small differences may arise due to the use of different integration routines, numerical precision schemes *et cetera* (P Kernan, private communication) and provide an estimate of the systematic computational uncertainty. This should be borne in mind when discussing the helium abundance to the "third decimal place".†

† Note that the $Y_{\rm p}$ values in Kernan and Krauss (1994) are not, as stated therein, higher by 0.003 than those in Walker *et al* (1991) and Smith *et al* (1993), all of whom used $\tau_{\rm n} = 889$ sec; in fact they are higher by only about half that amount, presumably just due to the incorporation of Seckel's (1993) nucleon mass correction.

Our best fits for the other elemental abundances over the range $\eta = 10^{-10} - 10^{-9}$, together with the typical errors, are:

$$\begin{aligned}
\left(\frac{\mathrm{D}}{\mathrm{H}}\right)_{\mathrm{p}} &= 3.6 \times 10^{-5\pm0.06} \left(\frac{\eta}{5\times10^{-10}}\right)^{-1.6} , \\
\left(\frac{^{3}\mathrm{He}}{\mathrm{H}}\right)_{\mathrm{p}} &= 1.2 \times 10^{-5\pm0.06} \left(\frac{\eta}{5\times10^{-10}}\right)^{-0.63} , \\
\left(\frac{^{7}\mathrm{Li}}{\mathrm{H}}\right)_{\mathrm{p}} &= 1.2 \times 10^{-11\pm0.2} \left[\left(\frac{\eta}{5\times10^{-10}}\right)^{-2.38} + 21.7\left(\frac{\eta}{5\times10^{-10}}\right)^{2.38}\right] .
\end{aligned} \tag{3.51}$$

These are in excellent agreement with the values obtained by Kernan and Krauss (1994) and Krauss and Kernan (1995). (The error band for ^{7}Li obtained by Monte Carlo is actually $\approx 10\%$ wider at $\eta \lesssim 2\times10^{-10}$ than is indicated above, as seen in figure 5.)

We now proceed to discuss the observed elemental abundances and their inferred primordial values which we can compare with the model predictions shown in figure 5.

3.2. Primordial elemental abundances

As mentioned earlier, the comparison of the predicted elemental abundances with observational data is complicated by the fact that the primordial abundances may have been significantly altered during the lifetime of the universe by nuclear processing in stars. Moreover this happens differently for different elements, for example ^{4}He, a very stable nucleus, grows in abundance with time as it is synthesized in stars, while D, which is very weakly bound, is always destroyed in stars. The history of ^{3}He and ^{7}Li is more complicated since these elements may be both created and destroyed through stellar processing. Whenever possible, astronomers endeavour to measure light element abundances in the most primordial material available and the recent development of large telescopes and CCD imaging technology have led to significant advances in the field. Pagel (1982, 1987, 1992) and Boesgaard and Steigman (1985) have comprehensively reviewed the observational data and discussed how the primordial abundances may be inferred by allowing for the effects of stellar evolution and galactic chemical evolution. The interested reader is urged to refer to the original literature cited therein to appreciate the uncertainties involved, both in the measurement of cosmic abundances today and in the bold astrophysical modelling necessary to deduce their values over 10 Gyr ago (1 Gyr $\equiv 10^{9}$ yr). Subsequently there have been several observational developments, some of which have been discussed by Pagel (1993). We review the key results which may be used to confront the standard BBN model.

3.2.1. Helium-4: The most important primordially synthesized element, ^{4}He, has been detected, mostly through its optical line emission, in a variety of astrophysical environments, e.g. planetary atmospheres, young stars, planetary nebulae and emission nebulae — galactic as well as extragalactic (see Shaver *et al* 1983), as well as in the intergalactic medium at a redhift of $z \simeq 3.2$ (Jakobsen *et al* 1994). Hence there is no doubt about the existence of an "universal" helium abundance of $\approx 25\%$ by mass. However, helium is also manufactured in stars, hence to determine its primordial abundance we must allow for the stellar helium component through its correlation with some other element, such as nitrogen or oxygen, which is made only in stars (Peimbert and Torres-Peimbert 1974). This is best done by studying recombination lines from HII regions in blue compact galaxies (BCGs) where relatively little stellar activity has occured, as evidenced by their low 'metal' abundance. (This refers, in astronomical jargon, to any element heavier than helium!)

In figure 6(a) we show a correlation plot of the observed helium abundance against that of oxygen and nitrogen as measured in 33 such selected objects (Pagel *et al* 1992). A linear trend is suggested by the data and extrapolation to zero metal abundance yields the primordial helium abundance with a small statistical error, $Y_{\rm p}(^4{\rm He}) = 0.228 \pm 0.005$.† It has however been emphasized, particularly by Davidson and Kinman (1985), that there may be large *systematic* errors in these abundance determinations, associated with corrections for (unobservable) neutral helium, underlying stellar helium absorption lines, collisional excitation *et cetera* ; these authors suggested, in common with Shields (1987), that the systematic error in $Y_{\rm p}$ could be as high as ± 0.01. The recent work by Pagel *et al* (1992) has specifically addressed several such sources of error; for example attention is restricted to objects where the ionizing stars are so hot that the correction for neutral helium is negligible. Hence these authors believe that the systematic error has now been reduced to about the same level as the statistical error, i.e.

$$Y_{\rm p}(^4{\rm He}) = 0.228 \pm 0.005\,({\rm stat}) \pm 0.005\,({\rm syst}) \ . \tag{3.52}$$

Since the (uncertain) systematic error is correlated between the different data points rather than being random, we cannot assign a formal confidence level to a departure from the mean value. It is common practice nonetheless to simply add the errors in quadrature. Another method is to add the systematic error to the adopted result and then deduce a 95% c.l. bound from the statistical error, assuming a Gaussian distribution (Pagel *et al* 1992); this gives the bounds

$$0.214 < Y_{\rm p}(^4{\rm He}) < 0.242 \ (95\% \ {\rm c.l.}) \ . \tag{3.53}$$

Mathews *et al* (1993) have suggested that galactic chemical evolution causes the correlation between helium and nitrogen to be non-linear at low metallicity, consequently the extrapolated helium abundance at zero metallicity is subject to an upward bias. Their own fits to the data, based on chemical evolution arguments, yield the same

† The χ^2 per degree of freedom for this fit is only 0.3, suggesting that the quoted statistical measurement errors (typically $\pm 4\%$) may have been overestimated (Pagel *et al* 1992).

value as Pagel *et al* (1992) for the regression with oxygen, but a lower value $Y_{\rm p}(^4{\rm He}) = 0.223 \pm 0.006$ for the regression with nitrogen.‡ However Pagel and Kazlauskas (1992) argue that the observed constancy of the N/O ratio at low metallicity favours the linear extrapolation used by Pagel *et al* (1992). Nevertheless, it *is* a matter of concern that the observed slope of the regression against oxygen, $\mathrm{d}Y/\mathrm{d}Z \approx 6$, is several times higher than the value expected from general chemical evolution arguments (see Pagel 1993). Other potential problems concern recent observational claims of low abundances in individual metal-poor galaxies, e.g. $Y(^4{\rm He}) = 0.216 \pm 0.006$ in *SBS 0335-052* (Melnick *et al* 1992). This particular result is unreliable on technical grounds, viz. underlying stellar absorption at $\lambda\,4471$ (Pagel 1993); however it is important that such objects be investigated further. In fact Skillman and Kennicutt (1993) have recently obtained

$$Y(^4{\rm He}) = 0.231 \pm 0.006 \ , \tag{3.54}$$

in *IZw18*, the most metal-poor galaxy known, in agreement with the primordial value derived by Pagel *et al* (1992). Also, observations of 11 new metal-poor BCGs by Skillman *et al* (1993) yield a similar result. A combined fit to these data together with that of Pagel *et al* gives

$$Y_{\rm p}(^4{\rm He}) = 0.232 \pm 0.003 \ . \tag{3.55}$$

after some "discrepant" objects are excluded (Olive and Steigman 1995a).

An important question is whether the above analyses have failed to identify any other systematic errors in the extraction of $Y_{\rm p}$. For example, all the observers cited above use the helium emissivities given by Brocklehurst (1972). Skillman and Kennicutt (1993) note that use of the emissivities of Smits (1991) would raise the derived value (3.54) in *IZw18* to 0.238 and the corresponding 2σ upper bound to 0.25. However the Smits (1991) emissivities were themselves erroneous and have been corrected by Smits (1996). Sasselov and Goldwirth (1994) have emphasized that use of the Smits (1996) emissivities gives a better fit to detailed line ratios than the Brocklehurst (1972) emissivities which are known to have problems with the fluxes of the triplet HeI lines used to extract $Y_{\rm p}$. These authors argue that consideration of additional systematic effects such as inadequacies in the ('Case B') radiative transfer model used and correction for neutral helium may raise the upper bound on $Y_{\rm p}$ to 0.255 for the data set analysed by Olive and Steigman 1995a), and as high as 0.258 for the measurements of *IZw18* by Skillman and Kennicutt (1993).

‡ These authors state (see also Fuller *et al* 1991) that this value is "... 2σ below the *lower* bound, $Y_{\rm p} > 0.236$, allowed in the standard BBN model with three neutrino flavours" (as quoted by Walker *et al* 1991) and suggest various modifications to the model to resolve the discrepancy. In fact the expected helium abundance in the standard BBN model can be much lower (see figure 5) if the uncertain upper bound (3.67) on $\rm D + {}^3He$ is ignored.

This question was examined in a study of 10 additional low metallicity BCGs by Izotov *et al* (1994). Whereas use of the emissivities of Brocklehurst (1972) yields $Y_\mathrm{p}(^4\mathrm{He}) = 0.229 \pm 0.004$ in excellent agreement with previous results (e.g. Pagel *et al* 1992), use of the Smits (1996) emissivities raises the value to $Y_\mathrm{p}(^4\mathrm{He}) = 0.240 \pm 0.005$. Izotov *et al* (1996) have recently increased their data sample to 27 HII regions in 23 BCGs and obtain†

$$Y_\mathrm{p}(^4\mathrm{He}) = 0.243 \pm 0.003 \ . \tag{3.56}$$

It is seen from figure 6(b) that use of the new emissivities (together with the new correction factors (Kingdon and Ferland 1995) for the collisional enhancement of He I emission lines) also decreases the dispersion of the data points in the regression plots and the derived slope, $\mathrm{d}Y/\mathrm{d}Z \approx 1.7 \pm 0.9$, is now smaller than found before (e.g. by Pagel *et al* 1992) and in agreement with general expectations in chemical evolution models. Systematic effects due to deviations from Case B recombination theory, temperature fluctuations, Wolf-Rayet stellar winds and supernova shock waves are demonstrated to be negligible while different corrections for underlying stellar absorption and fluorescent enhancement in the HeI lines alter Y_p by at most ± 0.001. (For example, use of the older collisional enhancement correction factors (Clegg 1983) gives $Y_\mathrm{p} = 0.242 \pm 0.004$.) Thus, while the widely adopted bound $Y_\mathrm{p}(^4\mathrm{He}) < 0.24$ (e.g. Walker *et al* 1991, Smith *et al* 1993, Kernan and Krauss 1994) based on (3.53) may be "reasonable", a more "reliable" upper bound to the primordial helium abundance is (Kernan and Sarkar 1996)

$$Y_\mathrm{p}(^4\mathrm{He}) < 0.25 \ . \tag{3.57}$$

Krauss and Kernan (1995) also consider a value of Y_p as large as 0.25. Copi *et al* (1995a) favour a "reasonable" bound of 0.243 and an "extreme" bound of 0.254.

3.2.2. Deuterium: The primordial abundance of deuterium is even harder to pin down since it is easily destroyed in stars at temperatures exceeding $\approx 6 \times 10^5\,\mathrm{K}$); in fact, its spectral lines have not been detected in any star, implying $\mathrm{D/H} < 10^{-6}$ in stellar atmospheres. It is seen in the giant planets, which reflect the composition of the pre-Solar nebula, with an abundance $\mathrm{D/H} \approx (1-4) \times 10^{-5}$. It is also detected in the local interstellar medium (ISM) through its ultraviolet absorption lines in stellar spectra but its abundance shows a large scatter, $\mathrm{D/H} \approx (0.2-4) \times 10^{-5}$, suggesting localized abundance fluctuations and/or systematic errors. Even among the cleanest lines of sight (towards hot stars within about 1 kpc) the abundance as measured by the *Copernicus* and *IUE* satellites varies in the range $\mathrm{D/H} \approx (0.8-2) \times 10^{-5}$ (Laurent 1983,

† The central value is now higher by 0.002 than the result $Y_\mathrm{p}(^4\mathrm{He}) = 0.241 \pm 0.003$ quoted by Thuan *et al* (1996) because the most metal-deficient BCG (*I Zw 18*) has now been excluded from the sample on account of its abnormally low HeI line intensities (see discussion in Izotov *et al* 1996).

Vidal-Madjar 1986). From a careful analysis of the available data, McCullough (1992) finds that after discarding some unreliable measurements, the remaining 7 *IUE* and 14 *Copernicus* measurements are all consistent with an interstellar abundance of

$$\left(\frac{\mathrm{D}}{\mathrm{H}}\right)_{\mathrm{ISM}} = 1.5 \pm 0.2 \times 10^{-5} \ . \tag{3.58}$$

Linsky *et al* (1993, 1995) have measured $\mathrm{D/H} = 1.60 \pm 0.09\,(\mathrm{stat})\,^{+0.05}_{-0.10}\,(\mathrm{syst}) \times 10^{-5}$ towards the star *Capella* at 12.5 kpc. using the *Hubble Space Telescope* (see figure 7). However since the Lyman-α line (of hydrogen) is severely saturated even towards such a nearby star, such observations, although very accurate, cannot test whether there are real spatial variations in the interstellar deuterium abundance (Pagel 1993). Also the entire data set is still too limited to reveal any correlation of the D abundance with the metallicity (Pasachoff and Vidal-Madjar 1989).

It has been argued that there are no important astrophysical sources of deuterium (Epstein *et al* 1976) and ongoing observational attempts to detect signs of deuterium synthesis in the Galaxy have so far not contradicted this belief (see Pasachoff and Vidal-Madjar 1989). If this is indeed so, then the lowest D abundance observed today should provide a *lower* bound to the primordial abundance. McCullough's (1992) analysis of the observations discussed above then implies:

$$\left(\frac{\mathrm{D}}{\mathrm{H}}\right)_{\mathrm{p}} > 1.1 \times 10^{-5} \ (95\% \ \mathrm{c.l.}). \tag{3.59}$$

which we consider a "reliable" bound (Kernan and Sarkar 1996).

To obtain an upper bound to the primordial abundance, it has been traditional to resort to models of galactic chemical evolution which indicate that primordial D has been depleted due to cycling through stars ('astration') by a factor of about $2-10$ (e.g. Audouze and Tinsley 1976, Clayton 1985, Delbourgo-Salvador *et al* 1985, Vangioni-Flam *et al* 1994). The depletion factor may be, moreover, variable within the Galaxy (e.g. Delbourgo-Salvador *et al* 1987), leading to large fluctuations in the observed interstellar abundance today; unfortunately, as mentioned above, this hypothesis cannot be observationally tested. Hence by these arguments the primordial abundance of deuterium is very approximately bounded to be less than a few times 10^{-4}. It is obviously crucial to detect deuterium outside the Solar system and the nearby interstellar medium in order to get at its primordial abundance and also, of course, to establish its cosmological origin. Adams (1976) had proposed searching for Lyman-series absorption lines of deuterium in the spectra of distant quasars, due to foreground intergalactic clouds made of primordial unprocessed material. However problems arise in studying such quasar absorption systems (QAS) because of possible confusion with neighbouring absorption lines of hydrogen and multi-component velocity structure in the clouds (Webb *et al* 1991). The recent availability of large aperture ground-based telescopes, e.g. the 10-mt *Keck Telescope*, has provided the required sensitivity and spectral resolution, leading to several detections. Songaila *et al* (1994) find

$$\left(\frac{\mathrm{D}}{\mathrm{H}}\right)_{\mathrm{QAS(1)}} \approx (1.9 - 2.5) \times 10^{-4} \ , \tag{3.60}$$

in a chemically unevolved cloud at $z = 3.32$ along the line of sight to the quasar *Q0014+813*, and note that there is a 3% probability of the absorption feature being a misidentified Ly-α line of hydrogen. Carswell *et al* (1994) obtain $\mathrm{D/H} = 10^{-3.6\pm0.3}$ in the same cloud but the confusion probability in their data is said to be as high as 15%. However, further observations have resolved D lines at $z = 3.320482$ and $z = 3.320790$, thus eliminating the possibility of such confusion; the measured abundances in the two clouds are, respectively, $\mathrm{D/H} = 10^{-3.73\pm0.12}$ and $10^{-3.72\pm0.09}$ (where the errors are *not* gaussian) (Rugers and Hogan 1996a). These authors also set an independent lower limit of $\mathrm{D/H} \geq 1.3 \times 10^{-4}$(95% c.l.) on their sum from the Lyman limit opacity. Recently, they have detected $\mathrm{D/H} = 1.9^{+0.6}_{-0.9} \times 10^{-4}$ in another QAS at $z = 2.797957$ towards the same quasar; The errors are higher because the D feature is saturated, even so a 95% c.l. lower limit of $\mathrm{D/H} > 0.7 \times 10^{-4}$ is obtained (Rugers and Hogan 1996b). There have been other, less definitive, observations of QAS consistent with this abundance, e.g. $\mathrm{D/H} \approx 10^{-3.95\pm0.54}$ at $z = 2.89040$ towards *GC0636+68* (Hogan 1995a), $\mathrm{D/H} \lesssim 1.5 \times 10^{-4}$ at $z = 4.672$ towards *BR1202-0725* (Wampler *et al* 1995) and $\mathrm{D/H} \lesssim 10^{-3.9\pm0.4}$ at $z = 3.08$ towards *Q0420-388* (Carswell *et al* 1996).† However, very recently, other observers have found much lower values in QAS at $z = 3.572$ towards *Q1937-1009* (Tytler *et al* 1996) and at $z = 2.504$ towards *Q1009+2956* (Burles and Tytler 1996); their average abundance is

$$\left(\frac{\mathrm{D}}{\mathrm{H}}\right)_{\mathrm{QAS(2)}} = 2.4 \pm 0.3 \ (\mathrm{stat}) \pm 0.3 \ (\mathrm{syst}) \times 10^{-5} \ . \tag{3.61}$$

Unlike the cloud in which the abundance (3.60) was measured, these QAS also exhibit absorption due to C and Si, whose synthesis in stars would have been accompanied by destruction of D. Tytler *et al* (1996) argue that this must have been negligible since the metallicity is very low. Although this is true averaged over the cloud, large fluctuations in the observed D abundance are possible since the mass of absorbing gas covering the QSO image is very small; thus D may well have been significantly depleted in it by a star which was not massive enough to eject 'metals' (Rugers and Hogan 1996b). Tytler and Burles (1996) point out in response that the line of sight through the QAS is $\sim 10\,\mathrm{Kpc}$ long so it would be difficult for metals to be removed from two independent lines of sight, leaving the same D abundance in each. They also note that their data quality is superior to the other detections and upper limits. In view of this

† The metallicity in the last two QAS are, respectively, about 1/10 of Solar and 2 times Solar, showing that significant stellar processing has already occured even at such high redshifts!

confusing observational situation and keeping in mind that D is *always* destroyed by stellar processing, we adopt the high D measurement (3.60) in the chemically unevolved cloud as a conservative upper limit on its primordial abundance.

$$\left(\frac{\mathrm{D}}{\mathrm{H}}\right)_{\mathrm{p}} \lesssim 2.5 \times 10^{-4} , \tag{3.62}$$

although one cannot attach a confidence level to this number. Given the contradictory observations, it is probably premature to interpret (3.60) as providing a *lower* bound on the primordial deuterium abundance (cf. Krauss and Kernan 1995).

Edmunds (1994) has argued that a primordial abundance as large as in (3.60) cannot be reduced to the present ISM value (3.58) in a simple 'closed box' chemical evolution model. If so, the lower QAS measurement (3.61) would be favoured as the primordial one. However this chemical evolution model is well known to be inadequate in many respects (see Rana 1991) and cannot yield any reliable conclusions. A definitive resolution of the discrepancy between the two values can only come from further observations which are in progress

3.2.3. Helium-3: The abundance of ^{3}He is similarly uncertain, with the additional complication that it is capable of being both produced and destroyed in stars. It has been detected through its radio recombination line in galactic HII regions although initial attempts to measure its abundance gave rather widely varying results in the range $\approx (1-15) \times 10^{-5}$ along with some upper bounds at the $\approx 10^{-5}$ level (Bania *et al* 1987). Balser *et al* (1994) have recently made considerable progress in overcoming the observational problems involved in determining the rather weak line parameters and in modelling the HII regions; they now obtain more stable abundances in the range

$$\left(\frac{^3\mathrm{He}}{\mathrm{H}}\right)_{\mathrm{HII}} \approx (1-4) \times 10^{-5} , \tag{3.63}$$

in a dozen selected regions. Also Rood *et al* (1992) have detected a large abundance ($^3\mathrm{He/H} \approx 10^{-3}$) in the planetary nebula *NGC 3242*, in accord with the expectation (Rood *et al* 1976, Iben and Truran 1978) that stars comparable in mass to the Sun create ^{3}He. However massive stars destroy ^{3}He, hence to determine the overall consequence of astration requires detailed modelling of stellar evolution and averaging over some assumed initial mass function (IMF) of stars. A detailed study by Dearborn *et al* (1986) considered several possibilities for the initial helium and 'metal' abundances and averaged over a Salpeter type IMF: $\mathrm{d}N/\mathrm{d}M \propto M^{-1.35}$ (see Scalo 1986). These authors found that the net fraction, g_3, of ^{3}He which survives stellar processing is quite sensitive to the assumed initial abundances. When averaged over stars of mass $3\,M_\odot$ and above, g_3 is as large as 0.47 for a standard Pop I composition (28% helium, 2% metals) but as small as 0.04 for an extreme low metal model (25% helium, 0.04%

metals). However if stars of mass down to $0.8\,M_\odot$ are included, then g_3 exceeds 0.5 for any assumed composition.† If however there has indeed been *net* creation of ^{3}He in stars, it is puzzling that the galactic observations find the highest ^{3}He abundances in the outer Galaxy where stellar activity is *less* than in the inner Galaxy. While regions with high abundances do lie preferentially in the Perseus spiral arm, there are large source-to-source variations which do not correlate with stellar activity (Balser *et al* 1994). Secondly, in this picture the present day interstellar ^{3}He abundance should be significantly higher than its proto-Solar abundance as measured in meteorites (3.65); however several of the ISM abundances are *less* than the Solar system value.

To reconcile these discrepancies, Hogan (1995b) has suggested that there may in fact be net *destruction* of ^{3}He in $\approx 1-2\,M_\odot$ stars through the same mixing process which appears to be needed to explain other observations, e.g. the ^{12}C/^{13}C ratio (Wasserburg *et al* 1995). A plausible mechanism for this has also been suggested (Charbonnel 1995). This is indeed essential if the primordial deuterium abundance is as high as is indicated by the recent measurement (3.60) in a QAS. Although the D can be astrated down to its much lower abundance in the ISM, the ^{3}He thus produced would exceed observational bounds unless it too is destroyed to a large extent. These considerations have prompted reexamination of the usual assumptions about the chemical evolution of ^{3}He (e.g. Vangioni-Flam and Cassé 1995, Copi *et al* 1995c, Galli *et al* 1995, Olive *et al* 1995, Palla *et al* 1995, Scully *et al* 1996). It is clear that until this is is better understood, one cannot use the ^{3}He abundance to sensibly constrain BBN (cf. Wilson and Rood 1994).

3.2.4. Deuterium + Helium-3: Yang *et al* (1984) had suggested that the uncertainties in determining the primordial abundances of D and ^{3}He may be circumvented by considering their *sum*. Since D is burnt in stars to ^{3}He, a fraction g_3 of which survives stellar processing, the primordial abundances may be related to the abundances later in time in a simple 'one-cycle' approximation to galactic chemical evolution. Neglecting the possible production of ^{3}He in light stars yields the inequality

$$\left(\frac{\mathrm{D}+{}^3\mathrm{He}}{\mathrm{H}}\right)_{\mathrm{p}} < \left(\frac{\mathrm{D}+{}^3\mathrm{He}}{\mathrm{H}}\right) + \left(\frac{1}{g_3}-1\right)\left(\frac{{}^3\mathrm{He}}{\mathrm{H}}\right) . \qquad (3.64)$$

The terms on the RHS may be estimated at the time of formation of the Solar system, about 4.5 Gyr ago, as follows. The abundance of ^{3}He in the Solar wind, deduced from studies of gas-rich meteorites, lunar rocks and metal foils exposed on lunar missions, may be identified with the sum of the pre-Solar abundances of ^{3}He and D (which was burnt to ^{3}He in the Sun), while the smallest ^{3}He abundance found in carbonaceous

† This is presumably because stars in the mass range $0.8-3\,M_\odot$, which contribute dominantly to the average over the assumed power law mass function, were assumed not to destroy *any* ^{3}He.

chondrites, which are believed to reflect the composition of the pre-Solar nebula, may be identified with the pre-Solar abundance of ^{3}He alone (Black 1971, Geiss and Reeves 1972). Such abundance determinations are actually made in ratio to ^{4}He; combining these data with the standard Solar model estimate $(^4\mathrm{He/H})_\odot = 0.10 \pm 0.01$ (see Bahcall and Ulrich 1988), Walker *et al* (1991) obtain:

$$1.3\times10^{-5} \lesssim \left(\frac{^3\mathrm{He}}{\mathrm{H}}\right)_\odot \lesssim 1.8\times10^{-5}\,, \qquad 3.3\times10^{-5} \lesssim \left(\frac{\mathrm{D}+{}^3\mathrm{He}}{\mathrm{H}}\right)_\odot \lesssim 4.9\times10^{-5}. \tag{3.65}$$

Although these are quoted as "2σ" bounds, we have chosen to view these as approximate inequalities since these authors have not estimated the systematic uncertainties. Walker *et al* also interpret the work by Dearborn *et al* (1986) on the survival of ^{3}He in stars to imply the lower limit

$$g_3 \gtrsim 0.25\,. \tag{3.66}$$

Using these values in (3.64) then bounds the primordial sum of D and ^{3}He as (Yang *et al* 1984, Walker *et al* 1991)

$$\left(\frac{\mathrm{D}+{}^3\mathrm{He}}{\mathrm{H}}\right)_\mathrm{p} \lesssim 10^{-4}\,. \tag{3.67}$$

(When confronting low nucleon density models, with $\eta \lesssim 2\times10^{-10}$, we may view this as essentially an upper bound on primordial D since the relative abundance of ^{3}He is then over a factor of 10 smaller.) Olive *et al* (1990) obtained a similar bound in the 'instantaneous recycling' approximation, i.e. assuming that some fraction of gas undergoes several cycles of stellar processing instantaneously. Steigman and Tosi (1992, 1995) obtain even stronger bounds in more elaborate models of galactic chemical evolution, for particular choices of the initial mass function, star formation rate, matter infall rate *et cetera*. Combining these results with the Solar system abundances discussed above, Hata *et al* (1996a) find the "primordial" abundances

$$1.5\times10^{-5} \leq \left(\frac{\mathrm{D}}{\mathrm{H}}\right)_\mathrm{p} \leq 10^{-4}, \qquad \left(\frac{^3\mathrm{He}}{\mathrm{H}}\right)_\mathrm{p} \leq 2.6\times10^{-5}. \tag{3.68}$$

In our opinion, all these bounds should be viewed with caution given the many astrophysical uncertainties in their derivation. Indeed there are now several observational indications that the bound (3.67) (or its minor variants) used in many recent analyses (e.g. Walker *et al* 1991, Smith *et al* 1993, Kernan and Krauss 1994, Copi *et al* 1995a, Krauss and Kernan 1995) is overly restrictive, and the bounds (3.68) used by Hata *et al* (1995) even more so. Geiss (1993) has recently reassessed the Solar system data and quotes more generous errors in the derived abundances:

$$\left(\frac{^3\mathrm{He}}{\mathrm{H}}\right)_\odot = 1.5\pm1.0\times10^{-5}\,, \qquad \left(\frac{\mathrm{D}+{}^3\mathrm{He}}{\mathrm{H}}\right)_\odot = 4.1\pm1.0\times10^{-5}\,. \tag{3.69}$$

If we use these numbers to estimate "2σ" limits, the bound (3.67) is immediately relaxed by a factor of 2! In fact it is not even clear if the Solar system abundances provide a representative measure at all, given that observations of ^{3}He elsewhere in the Galaxy reveal large (and unexplained) source-to-source variations (Balser *et al* 1994). Further, the survival fraction of ^{3}He may have been overestimated (Hogan 1995b). Note that if the primordial D abundance is as indeed as high as 2.5×10^{-4} (3.62), then not more than 10% of the ^{3}He into which it was burnt could have survived stellar processing, in conflict with the "theoretical" lower limit of 25% (3.66). Indeed, Gloeckler and Geiss (1996) have recently found, using the *Ulysses* spacecraft, that $^3\mathrm{He}/^4\mathrm{He} = 2.2^{+0.7}_{-0.6}\,(\mathrm{stat}) \pm 0.2\,(\mathrm{syst}) \times 10^{-4}$ in the local interstellar cloud. This is close to the value of $1.5 \pm 0.3 \times 10^{-4}$ in the pre-solar nebula (see (3.69)), demonstrating that the ^{3}He abundance has hardly increased since the formation of the solar system.

The Solar system abundances (3.65) imply $1.8 \times 10^{-5} \lesssim (\mathrm{D/H})_\odot \lesssim 3.3 \times 10^{-5}$, hence Walker *et al* (1991) as well as Smith *et al* (1993) adopt for the primordial value $(\mathrm{D/H})_\mathrm{P} \gtrsim 1.8 \times 10^{-5}$. We consider this to be less reliable than the lower bound (3.59) from direct observations of interstellar deuterium, particularly since Geiss (1993) quotes large errors from a reassessment of the Solar system data: $(\mathrm{D/H})_\odot = (2.6 \pm 1.0) \times 10^{-5}$. Copi *et al* (1995a) adopt a "sensible" lower bound of 1.6×10^{-5} while Krauss and Kernan (1995) opt for a lower bound of 2×10^{-5}.

3.2.5. Lithium-7: Lithium, whose common isotope is ^{7}Li, is observed in the atmospheres of both halo (Population II) and disk (Population I) stars, with widely differing abundances (see Michaud and Charbonneau 1991). This is not unexpected since ^{7}Li, like D, is easily destroyed (above 2×10^6 K) hence only the lithium on the stellar surface survives, to an extent dependent on the amount of mixing with the stellar interior, which in turn depends on the stellar temperature, rotation *et cetera* . For Pop I stars in open clusters with ages in the range $\sim 0.1 - 10$ Gyr, the observed abundances range upto $^7\mathrm{Li/H} \approx 10^{-9}$ (e.g. Hobbs and Pilachowski 1988). However in the somewhat older Pop II halo dwarfs, the abundance is observed to be about 10 times lower and, for high temperatures and low metallicity, nearly independent of the stellar temperature and the metal abundance (Spite and Spite 1982, Spite *et al* 1987, Rebolo *et al* 1988, Hobbs and Thorburn 1991). For a sample of 35 such stars with [Fe/H]< -1.3,† and $T \gtrsim 5500$ K and the weighted average of the lithium-7 abundance is (Walker *et al* 1991)

$$\left(\frac{^7\mathrm{Li}}{\mathrm{H}}\right)^{\mathrm{II}} = 10^{-9.92 \pm 0.07}\ (95\%\ \mathrm{c.l.})\ . \qquad (3.70)$$

† Square brackets indicate the logarithmic abundance relative to the Solar value, i.e. [Fe/H]< -1.3 means Fe/H$< 5 \times 10^{-2}(\mathrm{Fe/H})_\odot$.

This has been used to argue that the Pop II abundance reflects the primordial value in the gas from which the stars formed, with the higher abundance in the younger Pop I stars created subsequently, for example by supernovae (Dearborn *et al* 1989). Indeed evolutionary modelling (ignoring rotation) of halo stars indicate that they are essentially undepleted in lithium (Deliyannis *et al* 1990). Taking both observational errors and theoretical uncertainties (mostly the effects of diffusion) into account, these authors find the fitted initial abundance to be:

$$\left(\frac{^7\mathrm{Li}}{\mathrm{H}}\right)^{\mathrm{II}}_{\mathrm{p}} = 10^{-9.80\pm0.16}\ (95\%\ \mathrm{c.l.})\ . \tag{3.71}$$

This is assumed to be the primordial abundance by Walker *et al* (1991) and Smith *et al* (1993) since any production of lithium after the Big Bang, but before halo star formation is presumed to be unlikely (see Boesgaard and Steigman 1985). Kernan and Krauss (1994) also adopt this value.

Recently the situation has taken a new turn with the discovery that there are several extremely metal-poor Pop II halo dwarfs with *no* detectable lithium. Thorburn (1994) has determined accurate abundances for 80 stars with $[\mathrm{Fe/H}] < -1.9$ and $T > 5600\,\mathrm{K}$, of which 3 are lithium deficient with respect to the others by a factor of over 10 (see figure 8). Ignoring these reveals a weak, though statistically significant, trend of increasing ^{7}Li abundance with both increasing temperature and increasing metallicity which had not been apparent in older data (cf. Olive and Schramm 1992) but is also seen for a smaller sample of extreme halo dwarfs by Norris *et al* (1994). Thorburn (1994) interprets this as indicating lithium production by galactic cosmic ray spallation processes. Indeed beryllium (Gilmore *et al* 1992) and boron (Duncan *et al* 1992) have also been seen in several metal-poor halo stars with abundances *proportional* to the metallicity and in the ratio $\mathrm{B/Be} \approx 10$, which does point at such a production mechanism rather than a primordial origin. This should also have created lithium at the level of about 35% of its Pop II abundance; much of the observed dispersion about the Pop II 'plateau' would then be due to the $\approx 2\,\mathrm{Gyr}$ range in age of these stars. Thorburn (1994) therefore identifies the primordial abundance with the observed average value in the hottest, most metal-poor stars, viz.

$$\left(\frac{^7\mathrm{Li}}{\mathrm{H}}\right)^{\mathrm{II}}_{\mathrm{p}} = 10^{-9.78\pm0.20}\ (95\%\ \mathrm{c.l.})\ , \tag{3.72}$$

which agrees very well with the value (3.71) inferred by Deliyannis *et al* (1990). Subsequently, Thorburn's results have been questioned by Molaro *et al* (1995) who find *no* significant correlation of the lithium abundance, in a sample of 24 halo dwarfs, with either the temperature (when this is determined by a spectroscopic method rather than by broad-band photometry) or the metallicity (determined using an updated stellar atmosphere model). Thus they reaffirm the purely primordial origin of the Pop II ^{7}Li 'plateau' and argue that the observed dispersion is entirely due to measurement errors alone. However, Ryan *et al* (1996), who include data on 7 new halo dwarfs, have confirmed the original finding of Thorburn (1994); they note that Molaro *et al* (1995) did not test whether ^{7}Li is *simultaneously* correlated with T and [Fe/H]. In any case, the abundance Molaro *et al* (1995) derive for 24 stars with $T > 5700\,\mathrm{K}$ and [Fe/H]< -1.4 is fortuitously identical to that given by Thorburn (1994) so (3.72) remains the best estimate of the primordial Pop II ^{7}Li abundance. Even so this leaves open the question of why several stars which are in all respects similar to the other stars which define the Pop II 'plateau', are so lithium deficient. Until this is clarified, it may be premature to assert that the Pop II abundance of lithium reflects its primordial value.

Further, the observation that $^7\mathrm{Li/H} \approx 10^{-9}$ in Pop I stars as old as $\approx 10\,\mathrm{Gyr}$ (in *NGC 188*) then requires the galactic $^7\mathrm{Li/H}$ ratio to rise by a factor of about 10 in the first $\approx 2-5\,\mathrm{Gyr}$ and then remain constant for nearly 10 Gyr (Hobbs and Pilchaowski 1988). This encourages the *opposite* point of view, viz. that the (highest) Pop I abundance is that of primordially synthesized lithium, which has been (even more) depleted in the older Pop II stars, for example through turbulent mixing driven by stellar rotation (Vauclair 1988). The observational evidence for a $\pm 25\%$ dispersion in the Pop II ^{7}Li 'plateau' is consistent with this hypothesis (Deliyannis *et al* 1993). Rotational depletion was studied in detail by Pinsonneault *et al* (1992) who note that the depletion factor could have been as large as ≈ 10. Chaboyer and Demarque (1994) have demonstrated that models incorporating rotation provide a good match to the observed ^{7}Li depletion with decreasing temperature in Pop II stars and imply a primordial abundance

$$\left(\frac{^7\mathrm{Li}}{\mathrm{H}}\right)_{\mathrm{p}}^{\mathrm{I}} = 10^{-8.92\pm 0.1}, \tag{3.73}$$

corresponding to the highest observed Pop I value. (However the trend of increasing ^{7}Li abundance with increasing metallicity seen by Thorburn (1994) cannot be reproduced by these models.) Studies of galactic chemical evolution (Mathews *et al* 1990a, Brown 1992) show that both possibilities can be accomodated by the observational data, *including* the bound $^7\mathrm{Li/H} < 10^{-10}$ on the interstellar ^{7}Li abundance in the *Large Magellanic Cloud* along the line of sight to *Supernova 1987A* (e.g. Baade *et al* 1991). Although this apparently supports the Pop II abundance, the bound is considerably weakened by the uncertain correction for the depletion of lithium onto interstellar grains.

Recently Smith *et al* (1992) have detected ^{6}Li with an abundance

$$\left(\frac{^6\mathrm{Li}}{^7\mathrm{Li}}\right)^{\mathrm{II}} = 0.05 \pm 0.02\ , \tag{3.74}$$

in *HD 84937*, one of the hottest known Pop II stars. (Interestingly enough, to fit the observed spectrum requires line broadening of $\approx 5\,\mathrm{km\,sec^{-1}}$, suggestive of rotation.) Since ^{6}Li is much more fragile than ^{7}Li, this has been interpreted (e.g. Steigman *et* *al* 1993) as arguing against significant rotational depletion of primordially synthesized lithium since this would require the undepleted star to have formed with comparable amounts of ^{6}Li and ^{7}Li, whereas $^6\mathrm{Li}/^7\mathrm{Li} \approx 10^{-4}$ in standard BBN. The simplest interpretation is that the ^{6}Li (and some fraction of the ^{7}Li) was created by cosmic ray spallation processes. However this argument does not hold if there is some primordial source of ^{6}Li, as may happen in non-standard models.† Moreover Hobbs and Thorburn (1994) have found the same relative abundance of ^{6}Li in the cooler evolved subgiant *HD 201891*, which however has $^7\mathrm{Li/H} = 7.9 \times 10^{-11}$, a factor of 2 below the Pop II plateau, indicating that some depletion *has* occured. Vauclair and Charbonnel (1995) have pointed out that a mass loss of order $\sim 10^{-13} - 10^{-12}\, M_\odot\,\mathrm{yr}^{-1}$ through stellar winds can deplete ^{7}Li without depleting ^{6}Li; their preferred primordial abundance would then be the upper envelope of the Pop II value i.e. $(^7\mathrm{Li/H})_\mathrm{p} \approx 10^{-9.5\pm0.1}$.

Given these considerations, we believe the Pop I value (3.73) to be "reliable" and the new Pop II value (3.72) to be "reasonable" (Kernan and Sarkar 1996). Copi *et al* (1995a) consider an upper bound of $(^7\mathrm{Li/H})_\mathrm{p} \leq 3.5 \times 10^{-10}$ on the basis of the Pop II value, allowing for depletion by a factor of 2. Krauss and Kernan (1995) take the (older) Pop II value (3.71) to be primordial but also consider an upper bound as high as 5×10^{-10} to allow for some depletion.

3.3. Theory versus observations

We now determine the restrictions imposed on the nucleon-to-photon ratio by comparing the *inferred* bounds on the abundances of light elements with the 95% c.l. limits on their computed values. To begin with, we consider each element separately, as in previous work, although this procedure is, strictly speaking, statistically incorrect since the different elemental yields are correlated. Nevertheless it is an useful exercise to establish the approximate range of η for which there is concordance between the various abundances. First, consider the "reliable" abundance bounds (3.57), (3.59), (3.73):

$$
\begin{aligned}
& Y_\mathrm{p}(^4\mathrm{He}) < 0.25 && \Rightarrow \quad \eta < 9.1 \times 10^{-10} \,, \\
& \left(\frac{\mathrm{D}}{\mathrm{H}}\right)_\mathrm{p} > 1.1 \times 10^{-5} && \Rightarrow \quad \eta < 1.1 \times 10^{-9} \,, \\
& \left(\frac{^7\mathrm{Li}}{\mathrm{H}}\right)_\mathrm{p}^\mathrm{I} < 1.5 \times 10^{-9} && \Rightarrow \quad 4.1 \times 10^{-11} < \eta < 1.4 \times 10^{-9} \,.
\end{aligned}
\tag{3.75}
$$

† In this scenario protogalactic matter has been astrated by a large factor (Audouze and Silk 1989) implying that the primordial abundance of deuterium, another fragile isotope, should also be quite large, viz. $\mathrm{D/H} = (7 \pm 3) \times 10^{-4}$ (Steigman *et al* 1993). This is however not inconsistent with the recent direct observations of primordial deuterium (3.60), if the ^{3}He created by the astration of deuterium is also destroyed.

The "reasonable" abundance bounds (3.53), (3.62) and (3.72) yield:

$$\begin{aligned} Y_{\rm p}(^4{\rm He}) < 0.24 \quad &\Rightarrow \quad \eta < 3.4\times 10^{-10} , \\ \left(\frac{\rm D}{\rm H}\right)_{\rm p} \lesssim 2.5\times 10^{-4} \quad &\Rightarrow \quad \eta \gtrsim 1.3\times 10^{-10} , \\ \left(\frac{^7{\rm Li}}{\rm H}\right)_{\rm p}^{\rm II} < 2.6\times 10^{-10} \quad &\Rightarrow \quad 1.0\times 10^{-10} < \eta < 5.9\times 10^{-10} . \end{aligned} \tag{3.76}$$

Finally, the indirect bound (3.67)

$$\left(\frac{{\rm D}+{}^3{\rm He}}{\rm H}\right)_{\rm p} \lesssim 10^{-4} \quad \Rightarrow \quad \eta \gtrsim 2.6\times 10^{-10} , \tag{3.77}$$

provides a restrictive, albeit rather uncertain, lower limit on η. The situation is illustrated in figure 9 where we illustrate the consistency of standard BBN with the observations, viz. the primordial ^{4}He abundance (3.56) inferred from BCGs, the ISM D limit (3.59) and the two conflicting measurements (3.60) and (3.61) in QAS, and, finally, the ^{7}Li abundances in Pop II (3.72) and Pop I (3.73) stars.

3.3.1. Standard nucleosynthesis: Adopting the "reliable" bounds on extragalactic ^{4}He, interstellar D and Pop I ^{7}Li, we see from (3.75) that BBN can *conservatively* limit η to only within a factor of about 20:

$$4.1\times 10^{-11} < \eta < 9.1\times 10^{-10} \quad \Rightarrow \quad 0.0015 < \Omega_{\rm N}h^2 < 0.033 . \tag{3.78}$$

However this still improves on the observational uncertainty in η (3.31) by a factor of about 70. Note that the upper limit to η comes from ^{4}He, the element whose abundance is the *least* sensitive to the nucleon density. The one from interstellar D, which was historically crucial in establishing the consistency of BBN (Reeves *et al* 1973), is slightly less restrictive although arguably more robust and therefore still valuable. The Pop I ^{7}Li abundance provides a weak lower limit.

On the basis of the "reasonable" bounds quoted in (3.76), η can be pinned down to within a factor of about 3:

$$1.3\times 10^{-10} < \eta < 3.4\times 10^{-10} \quad \Rightarrow \quad 0.0048 < \Omega_{\rm N}h^2 < 0.013 , \tag{3.79}$$

i.e. assuming that the recent high D abundance measurement in a Lyman-α cloud bounds its primordial value and that the systematic error does not exceed the statistical error in the ^{4}He abundance determination. The Pop II ^{7}Li abundance provides a slightly less restrictive lower limit to η.

Finally, if we accept the upper bound on the sum of primordial D and ^{3}He inferred *indirectly* from Solar system abundances and stellar evolution arguments, then η is known (3.77) to within about 15% when combined with the "reasonable" upper bound on ^{4}He:

$$\eta \simeq (2.6-3.4)\times 10^{-10} \quad \Rightarrow \quad \Omega_{\rm N}h^2 \simeq 0.011\pm 0.0015 . \tag{3.80}$$

We emphasize that only this last constraint has been highlighted in the literature; for example, Smith *et al* (1993) quote $2.9 \leq (\eta/10^{10}) \leq 3.8$, corresponding to their adopted bounds $([\mathrm{D} + {}^3\mathrm{He}]/\mathrm{H})_\mathrm{p} \leq 9 \times 10^{-5}$ and $Y_\mathrm{p}({}^4\mathrm{He}) \leq 0.24$. Other groups have relied on ${}^7\mathrm{Li}$ rather than ${}^4\mathrm{He}$ to provide the upper bound to η, e.g. Walker *et al* (1991) adopt $({}^7\mathrm{Li}/\mathrm{H})_\mathrm{p}^\mathrm{II} \leq 1.4 \times 10^{-10}$ and quote $2.8 \leq (\eta/10^{-10}) \leq 4.0$, while Copi *et al* (1995a) adopt the more generous bounds $([\mathrm{D} + {}^3\mathrm{He}]/\mathrm{H})_\mathrm{p} \leq 1.1 \times 10^{-4}$ and $({}^7\mathrm{Li}/\mathrm{H})_\mathrm{p}^\mathrm{II} \leq 3.5 \times 10^{-10}$ to derive $2.5 \leq (\eta/10^{-10}) \leq 6.0$. In contrast, if primordial deuterium has indeed been detected with an abundance (3.60) of $\mathrm{D}/\mathrm{H} \simeq (1.9-2.5)\times10^{-4}$, then the implied nucleon density is about a factor of 2 smaller:†

$$\eta \simeq (1.3 - 1.9) \times 10^{-10} \quad \Rightarrow \quad \Omega_\mathrm{N} h^2 \simeq 0.0059 \pm 0.0011 \; , \tag{3.81}$$

while if the primordial abundance is instead in the 95% c.l. range $\mathrm{D}/\mathrm{H} \simeq (1.5 - 3.3) \times 10^{-5}$ (3.61), the implied nucleon density is about a factor of 2 bigger:

$$\eta = (4.6 - 8.1) \times 10^{-10} \quad \Rightarrow \quad \Omega_\mathrm{N} h^2 = 0.023 \pm 0.0064 \; (95\% \text{ c.l.}). \tag{3.82}$$

Of course both possiblities are consistent with the “reliable” bounds (3.78).

The above procedure of deriving limits on η using one element at a time ignores the fact that the different elemental yields are correlated. Taking this into account in a statistically consistent manner would lead to more stringent constraints than those obtained above using the symmetric 95% c.l. limits from the Monte Carlo procedure (Kernan and Krauss 1994). As seen in figure 10, the D abundance is strongly anti-correlated with the ${}^4\mathrm{He}$ abundance; hence those Monte Carlo runs in which the predicted ${}^4\mathrm{He}$ is lower than the mean, and which therefore may be allowed by some adopted observational upper bound, will also generally predict a higher than average D abundance, which may be in conflict with the corresponding observational upper bound. Krauss and Kernan (1995) determine the number of runs (as η is varied) which result in abundances simultaneously satisfying the upper bounds on ${}^4\mathrm{He}$, ${}^7\mathrm{Li}$ and $\mathrm{D} + {}^3\mathrm{He}$ and the lower bound on D. The maximum value of η is then found by requiring that 50 runs out of 1000 (upto $\sqrt{N}$ statistical fluctuations) satisfy all the constraints. Using their method and adopting the ISM bound (3.59) $\mathrm{D}/\mathrm{H} > 1.1 \times 10^{-5}$ and the Pop II bound (3.72) ${}^7\mathrm{Li}/\mathrm{H} < 2.6 \times 10^{-10}$, Kernan and Sarkar (1996) find that the maximum allowed value of η varies linearly with the adopted upper bound to the ${}^4\mathrm{He}$ as

$$\eta_\mathrm{max} = [3.19 + 375.7 \; (Y_\mathrm{p}^\mathrm{max} - 0.240)] \times 10^{-10} \; , \tag{3.83}$$

upto $Y_\mathrm{p}^\mathrm{max} = 0.247$; for higher Y_p, the Pop II ${}^7\mathrm{Li}$ bound (3.72) does not permit η to exceed 5.7×10^{-10}, as shown in figure 11 (a). If we choose instead to use the more

† Dar (1995) finds that a value of $\eta \simeq 1.6 \times 10^{-10}$ then provides the best fit (with a confidence level exceeding 70%) to the D, ${}^4\mathrm{He}$ and ${}^7\mathrm{Li}$ abundances, assuming $Y_\mathrm{p}({}^4\mathrm{He}) = 0.228 \pm 0.005$ and $({}^7\mathrm{Li}/\mathrm{H})_\mathrm{p}^\mathrm{II} = 1.7 \pm 0.4 \times 10^{-10}$. (Note however that his calculated abundances are systematically lower than in (3.50) and (3.51) and his adopted abundances differ from those given here.)

conservative Pop I bound (3.73) $^7\mathrm{Li/H} < 1.5 \times 10^{-9}$, the constraint is further relaxed to

$$\eta_{\max} = [3.28 + 216.4\,(Y_{\mathrm{p}}^{\max} - 0.240) + 34521\,(Y_{\mathrm{p}}^{\max} - 0.240)^2] \times 10^{-10} \ , \qquad (3.84)$$

for $Y_{\mathrm{p}} < 0.252$ and saturates at 1.06×10^{-9} for higher values, essentially due to the ISM D bound, as shown in figure 11 (b). Thus for $Y_{\mathrm{p}}^{\max} = 0.25$ (3.57), we find

$$\eta \leq 8.9 \times 10^{-10} \quad \Rightarrow \quad \Omega_{\mathrm{N}} h^2 \leq 0.033 \ , \qquad (3.85)$$

which is slightly more stringent than the value (3.75) determined using the (symmetric) 95% c.l. bounds on the ^{4}He abundance alone.

The above bounds on η were obtained assuming the validity of standard BBN amd may be altered in variant models as reviewed by Malaney and Mathews (1993). Below, we briefly discuss deviations which are permitted within the context of the Standard Model of particle physics † since we will discuss the effect of new physics, in section 4. We will also not consider the effect of gross departures from the standard cosmology, e.g. alternate theories of gravity and anisotropic world-models. In general, such deviations tend to speed up the expansion rate and increase the synthesized helium abundance, thus further tightening the constraints derived from standard BBN.

3.3.2. Inhomogeneous nucleosynthesis: The most well motivated departure from standard BBN is the possibility that nucleosynthesis occurs in an inhomogeneous medium, e.g. due to fluctuations generated by a first-order quark-hadron phase transition at $T_{\mathrm{c}}^{\mathrm{qh}} \approx 150-400\,\mathrm{MeV}$, a possibility emphasized by Witten (1984). As noted earlier (see footnote just before (3.36)) the signature for this would be the synthesis of significant amounts of elements beyond helium, although there is continuing controversy about the extent to which this would happen, due to the difficulty of adequately modelling the problem (e.g. Applegate *et al* 1988, Malaney and Fowler 1988, Terasawa and Sato 1991, Jedamzik *et al* 1994b). Observationally, there are no indications for an universal 'floor' in the abundances of such elements, particularly beryllium and boron, which would suggest a primordial origin (see Pagel 1993) and indeed their abundances are reasonably well understoood in terms of cosmic ray spallation processes (e.g. Prantzos *et al* 1993, Steigman *et al* 1993). Furthermore, recent theoretical developments suggest that the quark-hadron phase transition is effectively second-order (see Bonometto and Pantano 1993) and does not generate significant fluctuations in the

† As an exotic example of possibilities (far) beyond the SM, Bartlett and Hall (1991) have speculated that the comoving number of photons may *decrease* after the nuclosynthesis epoch if they become coupled to a cold 'hidden sector' via some mixing mechanism at a temperature of $\mathrm{O}(10)\,\mathrm{keV}$. Then the universe may indeed have the critical density in nucleons without violating the upper bound on the nucleon-to-photon ratio from BBN!

nucleon distribution (e.g. Banerjee and Gavai 1992). Nevertheless it is interesting to study the effect of hypothetical fluctuations on nucleosynthesis to see to what extent the standard picture may be altered. Such models (e.g. Kurki-Suonio *et al* 1990, Mathews *et al* 1990b, Jedamzik *et al* 1994a) can satisfy the conservative observational bounds (3.75) on ^{4}He, D and ^{7}Li (Pop I) with a higher nucleon density than in standard BBN; the upper limit to η is raised to

$$\eta \lesssim 2 \times 10^{-9} \quad \Rightarrow \quad \Omega_{\rm N} h^2 \lesssim 0.073 \, . \tag{3.86}$$

However the less reliable bound (3.72) on ^{7}Li (Pop II) and the indirect bound (3.67) on $(\mathrm{D} + {}^3\mathrm{He})/\mathrm{H}$ *are* violated unless η remains in about the same range (3.80) as is required by homogeneous nucleosynthesis on the basis of the same bounds. Thus even allowing for hypothetical and rather fine-tuned inhomogeneities, an Einstein-DeSitter universe with $\Omega_{\rm N} = 1$ is disfavoured. Although a nucleon-dominated universe which is open, e.g. having $\Omega_{\rm N} \approx 0.15$, is still allowed if one invokes inhomogeneous nucleosynthesis, there is no clear test of such a scenario. In particular the expected yields of 'r'-process elements (heavier than Si) is over a factor of 10^5 below presently observable bounds (Rauscher *et al* 1994).

3.3.3. 'Cascade' nucleosynthesis: In models with evaporating primordial black holes (Carlson *et al* 1990) (as also relic massive decaying particles (Dimopoulos *et al* 1988)), the nucleon density can be much higher with $\Omega_{\rm N} \approx 1$, since the photon and hadronic cascades triggered by the decay products (see section 4.2) can reprocess the excess ^{4}He and ^{7}Li and create acceptable amounts of D and ^{3}He for decay lifetimes in the range $\approx (2-9) \times 10^5$ sec.† The abundance of each element is determined by the fixed point of the balance equation incorporating its production by hadronic showers and destruction by photodissociation. The final ^{4}He abundance depends only on the product of the decaying particle abundance and baryonic branching ratio, while the other abundances are determined by the ratio of the particle mass to the baryonic branching ratio. The final abundance of ^{7}Li can be made consistent with either the Pop I or Pop II value but a large amount of ^{6}Li is also produced with $^6\mathrm{Li}/^7\mathrm{Li} \approx 3-10$, in apparent conflict with the

† Another way in which a decaying particle can allow a large nucleon density is if it creates non-thermal electron antineutrinos during nucleosynthesis with energies of O(MeV) which can convert protons into neutrons at late times. Thus enough D can be created (and not burnt further due to the low prevailing densities) while ^{7}Be (which would have subsequently decayed to overproduce ^{7}Li) is destroyed (Scherrer 1984, Terasawa and Sato 1987). However the increased expansion rate due to the decaying particle also boosts the neutron fraction at freeze-out, hence the final ^{4}He abundance. To allow $\Omega_{\rm N} h^2 \approx 0.2$ subject to the constraint $Y_{\rm p} \leq 0.25$ requires rather fine-tuned parameters e.g. a tau neutrino with $m_{\nu_\tau} \approx 20 - 30\,\mathrm{MeV}$, $\tau \approx 200 - 1000\,\mathrm{sec}$ and $m_{\nu_\tau} n_{\nu_\tau}/n_{\nu_{\rm e}} \approx 0.03 - 0.1\,\mathrm{MeV}$ (Gyuk and Turner 1994). Of course this possibility does not exist within the Standard Model and appears contrived even in extensions thereof since the decays must not create any visible energy (see section 5.1.2).

observed bound of $^6\mathrm{Li}/^7\mathrm{Li} \lesssim 0.1$ in Pop II stars. Dimopoulos *et al* (1988) have argued that since ^{6}Li is much more fragile than ^{7}Li, it may have been adequately depleted through rotational mixing (see Deliyannis *et al* 1990). Indeed ^{6}Li has been recently detected in two Pop II stars with an abundance (3.74) consistent with a primordial source, although admittedly there are difficulties in reconciling such a scenario with our present understanding of galactic chemical evolution (e.g. Audouze and Silk 1989, Steigman *et al* 1993).

The more modest aim of having a purely nucleonic universe with $\Omega_{\mathrm{N}} \approx 0.15$ can be achieved without (over)producing ^{6}Li in the scenario of Gnedin and Ostriker (1992) wherein an early generation of massive stars collapse to form black holes with accretion disks which emit high energy photons capable of photodissociating the overproduced helium and lithium. These authors confirm, as was noted earlier by Dimopoulos *et al* (1988), that reprocessing by photodisintegration alone cannot allow values higher than $\Omega_{\mathrm{N}} \approx 0.2$, contrary to the results of Audouze *et al* (1985).

3.3.4. Neutrino degeneracy: Finally we consider the possible role of neutrino degeneracy, which was first studied by Wagoner *et al* (1967). As mentioned earlier a chemical potential in electron neutrinos can alter neutron-proton equilibrium (3.39), as well as increase the expansion rate, the latter effect being less important. Consequently only the abundance of ^{4}He is significantly affected and the allowed range of η is still determined by the adopted primordial abundances of the other elements. For example, imposing $0.21 \leq Y_{\mathrm{P}} \leq 0.25$ then requires (e.g. Yahil and Beaudet 1976, David and Reeves 1980, Scherrer 1983, Terasawa and Sato 1988, Kang and Steigman 1992)

$$-0.06 \lesssim \xi_{\nu_{\mathrm{e}}} \lesssim 0.14 , \qquad (3.87)$$

assuming the chemical potential in other neutrino types, which can only increase the expansion rate, to be negligible. (For orientation, a value of $\xi_\nu \equiv \mu_\nu/T \approx \sqrt{2}$ is equivalent to adding an additional neutrino flavour (with $\xi_\nu = 0$) which we consider in section 4.1.) Now, if both $\xi_{\nu_{\mathrm{e}}}$ and $\xi_{\nu_{\mu,\tau}}$ are non-zero, then the lowering of the n/p ratio at freeze-out (due to $\xi_{\nu_{\mathrm{e}}}$) may be compensated for by the net speed-up of the expansion rate (due to $\xi_{\nu_{\mathrm{e},\mu,\tau}}$), thus enabling the ^{4}He and the D, ^{3}He abundances to be all within observational bounds even for large values of the nucleon density which are normally disallowed (Yahil and Beaudet 1976). Even the surviving ^{7}Li abundance, which is determined by a more complex interplay between reactions with different η dependence, may be made to match either its Pop I value (David and Reeves 1980) or its Pop II value (Olive *et al* 1991, Starkman 1992, Kang and Steigman 1992). For example, with $\xi_{\nu_{\mathrm{e}}} \approx 1.4 - 1.6$ *and* $\xi_{\nu_\mu,\nu_\tau} \approx 25 - 30$, one can have $\Omega_{\mathrm{N}} h^2$ as high as ≈ 1 (e.g. Starkman 1992). However such an universe would have been radiation dominated until well after the (re)combination epoch, making it difficult to create the observed large-scale structure (Freese *et al* 1983). Taking such constraints into account, Kang and Steigman (1992) quote the limits

$$-0.06 \lesssim \xi_{\nu_e} \lesssim 1.1\ , \quad |\xi_{\nu_\mu,\nu_\tau}| \lesssim 6.9\ , \quad \eta \lesssim 1.9\times 10^{-10}\ , \tag{3.88}$$

i.e. a critical density nucleonic universe is not permitted. In any case, earlier theoretical studies which allowed the possibility of generating such large lepton numbers (e.g. Langacker *et al* 1982) need to be reconsidered since we now know that ($B-L$ conserving) fermion number violation is unsuppressed even in the Standard Model at temperatures above the electroweak phase transition (see Shaposhnikov 1991, 1992). This would have converted part of any primordial lepton asymmetry into a baryon asymmetry, hence one cannot plausibly have $\xi_\nu \gg \eta$ without considerable fine-tuning (e.g. arranging for large cancellations between lepton asymmetries of opposite signs in different flavour channels). Therefore unless the lepton asymmetry is somehow generated *after* the electroweak phase transition, or unless the asymmetry is so large as to prevent the phase transition itself (see Linde 1979), it is reasonable to conclude that neutrino degeneracy cannot significantly affect the standard BBN model.

4. Constraints on new physics

Having established the consistency of standard BBN, we will now use it to constrain new physics beyond the Standard $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ Model. This has usually been done for specific models but one can identify two general classes of constraints, viz. those pertaining to stable particles (e.g. new massless neutrinos, goldstone bosons) which are relativistic during nucleosynthesis, and those concerned with massive, decaying particles (e.g. massive neutrinos, gravitinos) which are non-relativistic at this time. The extent to which we need to be cautious in this enterprise depends on the sensitivity of the physics under consideration to the light element abundances. For example, in constraining massive decaying particles (section 4.2), we can use together "reliable" as well as "indirect" bounds on elemental abundances, because the constraints can be simply scaled for different choices of the bounds. However in constraining the number of neutrino species or other light stable particles we must be more careful since the result is sensitive to the lower limit to η following from the abundance bounds on elements other than ^{4}He and cannot be simply scaled for different choices of such bounds.

4.1. Bounds on relativistic relics

The Standard Model contains only $N_\nu = 3$ weakly interacting massless neutrinos but in extensions beyond the SM there are often new superweakly interacting massless (or very light) particles. Since these do not usually couple to the Z^0 vector boson, there is no constraint on them from the precision studies at *LEP* of the 'invisible width' of

Z^0 decays which establish the number of $SU(2)_{\rm L}$ doublet neutrino species to be (LEP Elcetroweak Working Group 1995)

$$N_\nu = 2.991 \pm 0.016 \ , \tag{4.1}$$

and rule out any new particle with full strength weak interactions which has mass smaller than $\approx m_{Z^0}/2$. (Previous experiments which had set somewhat weaker bounds on N_ν are reviewed by Denegri *et al* (1990).)

Peebles (1966a) (see also Hoyle and Tayler 1964) had emphasized some time ago that new types of neutrinos (beyond the $\nu_{\rm e}$ and ν_μ then known) would increase the relativistic energy density, hence the expansion rate, during primordial nucleosynthesis, thus increasing the yield of ^{4}He. Subsequently Shvartsman (1969) pointed out that new superweakly interacting particles would have a similar effect and could therefore be constrained through the observational bound on the helium abundance. This argument was later quantified by Steigman *et al* (1977) for new types of neutrinos and by Steigman *et al* (1979) for new superweakly interacting particles. While the number of (doublet) neutrinos is now well known from laboratory experiments, the constraint on superweakly interacting particles from nucleosynthesis continues to be an unique probe of physics beyond the Standard Model and is therefore particularly valuable.

As we have seen earlier, increasing the assumed number of relativistic neutrino species N_ν increases g_ρ (2.71), hence the expansion rate (2.64), causing both earlier freeze-out with a larger neutron fraction (see (3.23)) and earlier onset of nucleosynthesis (see (3.34)), all together resulting in a larger value of $Y_{\rm p}(^4{\rm He})$ (see (3.49)).† One can parametrize the energy density of new relativistic particles in terms of the equivalent number N_ν of doublet neutrinos so that the limit on $\Delta N_\nu\,(\equiv N_\nu - 3)$ obtained by comparing the expected ^{4}He yield with its observational upper bound constrains the physics which determines the relic abundance of the new particles. (A complication arises if the tau neutrino is sufficiently massive so as to be non-relativistic during nucleosynthesis, as we will consider later; then the number of relativistic (doublet) neutrinos during nucleosynthesis would be 2 rather than 3.) The interaction rate keeping a superweakly interacting particle in thermal equilibrium will typically fall behind the Hubble expansion rate at a much higher 'decoupling' temperature than the value of a few MeV for (doublet) neutrinos. As discussed in section 2.2, if the comoving specific entropy increases afterwards, e.g. due to annihilations of massive particles in the Standard Model, the abundance of the new particles will be diluted relative to that of neutrinos, or equivalently their temperature will be lowered relatively (see (2.55)) since neutrinos in the SM have the same temperature as that of photons down to $T \sim m_{\rm e}$. Hence

† For a very large speed-up factor, there is no time for D and ^{3}He to be burnt to ^{4}He, hence $Y_{\rm p}$ begins to decrease with N_ν and drops below 25% for $N_\nu \gtrsim 6600$ (Ellis and Olive 1983). However this would overproduce D and ^{3}He by several orders of magnitude (Peebles 1971, Barrow 1983).

the energy density during nucleosynthesis of new massless particles i is equivalent to an effective number ΔN_ν of additional doublet neutrinos:

$$\Delta N_\nu = f_{\rm B,F} \sum_i \frac{g_i}{2} \left(\frac{T_i}{T_\nu}\right)^4 , \tag{4.2}$$

where $f_{\rm B} = 8/7$ (bosons) and $f_{\rm F} = 1$ (fermions), and T_i/T_ν is given by (2.55). Thus the number of such particles allowed by a given bound on ΔN_ν depends on how small T_i/T_ν is, i.e. on the ratio of the number of interacting relativistic degrees of freedom at $T_{\rm D}$ (when i decouples) to its value at a few MeV (when neutrinos decouple). Using table 1 we see that $T_{\rm D} > m_\mu$ implies $T_i/T_\nu < 0.910$ while $T_{\rm D} > T_{\rm c}^{\rm qh}$ bounds $T_i/T_\nu < 0.594$; the smallest possible value of T_i/T_ν in the Standard Model is 0.465, for $T_{\rm D} > T_{\rm c}^{\rm EW}$ (Olive *et al* 1981a).

For example, consider a new fermion F, e.g. a singlet (sterile) neutrino. Its decoupling temperature $T_{\rm D}$ can be approximately calculated, as for a doublet neutrino (2.67), by equating the interaction rate to the Hubble rate. For definiteness, consider annihilation to leptons and parametrize the cross-section as

$$\langle \sigma v \rangle_{\ell\bar{\ell}\to {\rm F}\,\bar{\rm F}} = \left(\frac{n_{\rm F}^{\rm eq}}{n_\ell^{\rm eq}}\right)^2 \langle \sigma v \rangle_{{\rm F}\,\bar{\rm F}\to \ell\bar{\ell}} \equiv \alpha T^2. \tag{4.3}$$

Then equating the annihilation rate $\Gamma_{\rm ann}^{\rm eq} = n_{\rm F} \langle \sigma v \rangle_{F\bar{F}\to \ell\bar{\ell}}$ to the expansion rate H (2.64) gives,

$$T_{\rm D} \simeq 7.2 \times 10^{-7}\ {\rm GeV}\, g_\rho^{1/6} \alpha^{-1/3}. \tag{4.4}$$

Thus the smaller the coupling, the earlier the particle decouples, e.g. $T_{\rm D} > m_\mu$ if $\alpha < 1.4 \times 10^{-15} {\rm GeV}^{-4}$,† and $T_{\rm D} > T_{\rm c}^{\rm qh}$ if $\alpha < 2.1 \times 10^{-16} {\rm GeV}^{-4} (T_{\rm c}^{\rm qh}/0.3\,{\rm GeV})^{-3}$. The energy density of the new particle during nucleosynthesis is then, respectively, equivalent to $\Delta N_\nu = 0.69$ and $\Delta N_\nu = 0.12$. Thus if the observationally inferred bound was, say $\Delta N_\nu < 1$, then one such singlet neutrino would be allowed per fermion generation only if they decoupled above $T_{\rm c}^{\rm qh}$. This requirement would impose an interesting constraint on the particle physics model in which such neutrinos appear.

We can now appreciate the significance of the precise bound on N_ν from nucleosynthesis. This depends on the adopted elemental abundances as well as uncertainties in the predicted values. Taking into account experimental uncertainties in the neutron lifetime and in nuclear reaction rates, the ^{4}He abundance can be calculated to within $\pm 0.5\%$ (see (3.50)). In contrast, the observationally inferred upper bound to $Y_{\rm p}$ is uncertain by as much as $\approx 4\%$ (compare (3.53) and (3.57)). More importantly, the bound on N_ν can only be derived if the nucleon-to-photon ratio η (or at least a lower bound to it) is known (see (3.49)). This involves comparison of the expected

† A more careful analysis of decoupling actually yields a less stringent bound (5.54).

and observed abundances of other elements such as D, ^{3}He and ^{7}Li which are much more poorly determined, both observationally and theoretically. It is a wide-spread misconception that the ^{4}He abundance *alone* constrains N_ν; in fact the effect of a faster expansion rate can be balanced by the effect of a lower nucleon density so that N_ν is *not at all constrained* for $\eta \lesssim 5\times10^{-11}$ which is quite consistent with the direct observational limit (3.31), as well as the reliable upper bound (3.75) to the ^{7}Li abundance! Of course with such a low nucleon density large amounts of D, ^{3}He and ^{7}Li would be created, in conflict with the "reasonable" observational bounds (3.76). Hence one can derive a lower limit to η from the abundances of these elements and then constrain N_ν given an observational upper bound on $Y_{\rm p}$. Thus the reliability of the BBN constraint on N_ν is essentially determined by the reliability of the lower limit to η, a fact that has perhaps not been always appreciated by particle physicists who use it to constrain various interesting extensions of the Standard Model. To emphasize this point, we briefly review the history of this constraint and comment in particular on those mentioned by the Particle Data Group (1992, 1994, 1996).

Steigman *et al* (1977) originally quoted the constraint $N_\nu \lesssim 7$ following from their assumption that $\Omega_{\rm N}h^2 \gtrsim 0.01$ (i.e. $\eta > 2.8 \times 10^{-10}$) and the conservative bound $Y_{\rm p} \leq 0.29$. Yang *et al* (1979) argued that a more restrictive bound $Y_{\rm p} \leq 0.25$ was indicated by observations and concluded that in this case *no* new neutrinos beyond $\nu_{\rm e}$, ν_μ and ν_τ were allowed. Their adopted limit $\Omega_{\rm N}h^2 \gtrsim 0.01$ was based on the assumption (following Gott *et al* 1974) that the dynamics of galaxies is governed by nucleonic matter. Following the growing realization that the dark matter in galaxies could in fact be non-baryonic, Olive *et al* (1981b) proposed a much weaker limit of $\eta > 2.9 \times 10^{-11}$ following from just the observed *luminous* matter in galaxies, and noted that *no* constraint on N_ν could then be derived for any reasonable bound on $Y_{\rm p}$. These authors presented a systematic analysis of how the inferred constraint on N_ν varies with the assumed nucleon density, neutron lifetime and ^{4}He abundance and emphasized the need for a detailed investigation of the other elemental abundances to better constrain η and N_ν.

This was done by Yang *et al* (1984) who proposed that the sum of primordial D and ^{3}He could be bounded by considerations of galactic chemical evolution; using Solar system data (3.65) they inferred $[({\rm D}+{}^3{\rm He})/{\rm H}]_{\rm p} \lesssim 10^{-4}$ and from this concluded $\eta \geq 3 \times 10^{-10}$ (see (3.77)). This yielded the often quoted constraint

$$N_\nu \leq 4 \; , \qquad (4.5)$$

assuming that $Y_{\rm p} \leq 0.25$ and $\tau_{\rm n} > 900\,$sec. Ellis *et al* (1986b) pointed out that this constraint could be relaxed since (a) laboratory experiments allowed for the neutron lifetime to be as low as 883 sec , (b) Possible systematic observational errors allowed for $Y_{\rm p}(^4{\rm He})$ to be as high as 0.26, and (c) the observational indication that there is net destruction of ^{3}He in stars (see discussion before (3.63)) allowed for $[({\rm D}+{}^3{\rm He})/{\rm H}]_{\rm p}$ to be as high as 5×10^{-4}. Thus *conservatively* BBN allows upto

$$N_\nu \lesssim 5.5 , \tag{4.6}$$

neutrinos. However Steigman *et al* (1986) reasserted the constraint in (4.5).

Subsequently precision measurements of the neutron lifetime (e.g. Mampe *et al* 1989) confirmed that it was lower than had been previously assumed. Moreover, Krauss and Romanelli (1990) quantified the uncertainties in the theoretical predictions by a Monte Carlo method taking all experimental uncertainties in input reaction rates into account. Combining these results with a detailed study of the ^{7}Li abundance evolution in halo stars, Deliyannis *et al* (1989) presented a new lower limit of $\eta > 1.2 \times 10^{-10}$ on the basis of the Pop II ^{7}Li observations. They noted that this would allow upto

$$N_\nu \leq 5 \tag{4.7}$$

neutrino species to be consistent with a primordial ^{4}He mass fraction less than 25%. Olive *et al* (1990) continued to adopt the indirect bound $[(\mathrm{D} + {}^3\mathrm{He})/\mathrm{H}]_\mathrm{p} \leq 1.1 \times 10^{-4}$ and assumed a more stringent upper bound $Y_\mathrm{p} \leq 0.24$ (3.53), so that their derived constraint on N_ν became even more restrictive

$$N_\nu \leq 3.4 . \tag{4.8}$$

Making nearly identical assumptions, viz. $\tau_\mathrm{n} > 882\,\mathrm{sec}$, $[(\mathrm{D} + {}^3\mathrm{He})/\mathrm{H}]_\mathrm{p} \leq 10^{-4}$, $Y_\mathrm{p}(^4\mathrm{He}) < 0.24$, Walker *et al* (1991) quoted an even tighter limit

$$N_\nu \leq 3.3 . \tag{4.9}$$

Subsequently, the possible detection of a large primordial deuterium abundance in a Lyman-α cloud (Songaila *et al* 1994, Rugers and Hogan 1996a,b), as well as observational indications that the helium-4 abundance may have been systematically underestimated (Sasselov and Goldwirth 1994, Izotov *et al* 1994, 1996) have further justified the caution advocated by Ellis *et al* (1986b) in deriving this important constraint. Nevertheless Copi *et al* (1995a) have recently reasserted the upper limit of 3.4 neutrino species, continuing to adopt similar bounds as before, viz. $\tau_\mathrm{n} > 885\,\mathrm{sec}$, $[(\mathrm{D} + {}^3\mathrm{He})/\mathrm{H}]_\mathrm{p} \leq 1.1 \times 10^{-4}$ and $Y_\mathrm{p}(^4\mathrm{He}) < 0.243$.

Kernan and Krauss (1994) have emphasized that the procedure used by all the above authors is statistically inconsistent since the abundances of the different elements are *correlated* and the use of symmetric confidence limits on the theoretical abundances is overly conservative. (Moreover, only Deliyanis *et al* (1989) allowed for errors in the expected yields due to reaction rate uncertainties.) Just as in the case of the derived limits on η (see discussion before (3.83)), a correct analysis allowing for correlations would yield a tighter constraint on N_ν. Kernan and Krauss (1994) illustrate this by considering the abundance bounds $[(\mathrm{D}+{}^3\mathrm{He})/\mathrm{H}]_\mathrm{p} \leq 10^{-4}$ and $Y_\mathrm{p}(^4\mathrm{He}) \leq 0.24$ advocated by Walker *et al* (1991) and determining the 95% c.l. limits on η and N_ν by requiring that at least 50 out of 1000 Monte Carlo runs lie within the *joint* range bounded by both $\mathrm{D}+{}^3\mathrm{He}$ and ${}^4\mathrm{He}$. As shown in figure 10 (a), this imposes tighter constraints than simply requiring that 50 runs lie, either to the left of the $(\mathrm{D}+{}^3\mathrm{He})$ bound (for low η), or below the ${}^4\mathrm{He}$ bound (for high η). Moreover, the procedure of simply checking whether the symmetric 95% c.l. limit for an individual elemental abundance is within the observational bound gives an even looser constraint. Figure 12(a) plots the number of Monte Carlo runs (out of 1000) which satisfy the joint observational bounds as a function of η for different values of N_ν; it is seen that the 95% c.l. limit is

$$N_\nu < 3.04 \ , \tag{4.10}$$

rather than 3.3 as quoted by Walker *et al* (1991).† As emphasized by Kernan and Krauss (1994), this is an extremely stringent constraint, if indeed true, on physics beyond the Standard Model. For example even a singlet neutrino which decouples above $T_{\mathrm{c}}^{\mathrm{EW}}$ will be equivalent to 0.047 extra neutrino species, and is therefore *excluded*. More crucially, the helium mass fraction (for N_ν=3) is now required to exceed 0.239 for consistency with the assumed bound on $\mathrm{D}+{}^3\mathrm{He}$, so a measurement below this value would rule out standard BBN altogether! Hence these authors draw attention again to the possibility that the systematic uncertainty in the usually quoted value (3.52) of $Y_{\mathrm{p}}({}^4\mathrm{He})$ has been underestimated. The N_ν limit can be simply parametrized as

$$N_\nu \leq 3.07 + 74.1\ (Y_{\mathrm{p}}^{\max} - 0.24) \ , \tag{4.11}$$

where $\tau_{\mathrm{n}} = 887 \pm 2\,\mathrm{sec}$ has been used (Krauss and Kernan 1995), so $N_\nu \leq 3.8$ for $Y_{\mathrm{p}}^{\max} = 0.25$ (3.57). Olive and Steigman (1995b) assign a low systematic error of ± 0.005 to their extrapolated primordial helium abundance $Y_{\mathrm{p}}({}^4\mathrm{He}) = 0.232 \pm 0.003$ (3.55) and thus obtain a best fit value of $N_\nu = 2.17 \pm 0.27\,(\mathrm{stat}) \pm 0.42\,(\mathrm{syst})$. This is unphysical if there are indeed at least 3 massless neutrinos so they compute the upper limit on N_ν restricting attention to the *physical* region alone (see Particle data Group 1996), obtaining

$$N_\nu < 3.6 \ . \tag{4.12}$$

They also impose the weaker condition $N_\nu \geq 2$ (as would be appropriate if the ν_τ was massive and decayed before nucleosynthesis) to obtain the bound $N_\nu < 3.2$.

As we have discussed earlier, the indirect bound $[(\mathrm{D}+{}^3\mathrm{He})/\mathrm{H}]_{\mathrm{p}} \leq 10^{-4}$ (3.67) used above is rather suspect and it would be more conservative to use the "reasonable" observational bounds $\mathrm{D/H} \lesssim 2.5\times10^{-4}$ (3.62) and $({}^7\mathrm{Li/H})_{\mathrm{p}}^{\mathrm{II}} \leq 2.6\times10^{-10}$ (3.72) to constrain η. A Monte Carlo exercise has been carried out for this case (Kernan and Sarkar 1996) and yields the constraint,

$$N_\nu \leq 3.75 + 78\ (Y_{\mathrm{p}}^{\max} - 0.24) \ , \tag{4.13}$$

† If correlations had not been included, the limit would have been 3.15, the difference from Walker *et al* being mainly due to the $\approx 1\%$ increase in the (more carefully) calculated ${}^4\mathrm{He}$ abundance.

if we require all constraints to be *simultaneously* satisfied. Thus, as shown in figure 12(b), the conservative limit is $N_\nu \leq 4.53$, if the ^{4}He mass fraction is as high as 25% (3.57). Equivalently, we can derive a limit on the 'speedup rate' of the Hubble expansion due to the presence of the additional neutrinos which contribute 7/4 each to g_ρ, the number of relativistic degrees of freedom (2.62), increasing it above its canonical value of 43/4 at this epoch. Then the time-temperature relationship (2.66) becomes modified as $t \to t' = \xi^{-1} t$, where

$$\xi \equiv \left[1 + \frac{7}{43}(N_\nu - 3)\right]^{1/2} . \quad (4.14)$$

Since ξ cannot far exceed unity, we obtain using (4.13),

$$\xi - 1 \lesssim 0.061 + 6.3\,(Y_{\rm p}^{\rm max} - 0.24) . \quad (4.15)$$

In contrast to our conservative approach, Hata *et al* (1995) deduce the even more restrictive values $({\rm D/H})_{\rm p} = 3.5^{+2.7}_{-1.8} \times 10^{-5}$ and $(^3{\rm He/H})_{\rm p} = 1.2 \pm 0.3 \times 10^{-5}$ ("95% c.l.") using a chemical evolution model normalized to Solar system abundances and convolving with BBN predictions (Hata *et al* 1996a). Combining this with the estimate $Y_{\rm p}(^4{\rm He}) = 0.232 \pm 0.003\,({\rm stat}) \pm 0.005\,({\rm syst})$ by Olive and Steigman (1995a), and adopting $(^7{\rm Li/H})_{\rm p} = 1.2^{+4.0}_{-0.5} \times 10^{-10}$ ("95% c.l.") these authors obtain $N_\nu = 2.0 \pm 0.3$. Thus they are led to conclude that $N_\nu < 2.6$ (95% c.l.) so the Standard Model ($N_\nu = 3$) is ruled out at the 98.6% c.l.! However the confidence levels quoted on their adopted elemental abundances are unreliable for the detailed reasons given in section 3.2, hence we believe this conclusion is not tenable.

The BBN bound on N_ν is under renewed discussion (e.g. Fields and Olive 1996, Fields *et al* 1996, Cardall and Fuller 1996a, Copi *et al* 1996, Hata *et al* 1996b) following the first direct measurements of deuterium at high redshifts which have called into question the chemical evolution models employed in earlier work (e.g. Yang *et al* 1984, Steigman *et al* 1986, Walker *et al* 1991). We consider the bounds given in (4.13) and (4.15) to be conservative and advocate their use by particle physicists seeking to constrain models of new physics.

4.2. Bounds on non-relativistic relics

The presence during nucleosynthesis of a *non-relativistic* particle, e.g. a massive neutrino, would also increase the energy density, hence the rate of expansion, and thus increase the synthesized abundances. This effect is however different from that due to the addition of a new relativistic particle, since the energy density of a non-relativistic particle decreases $\propto T^3$ (rather than $\propto T^4$) hence the speed-up rate due to the non-relativistic particle is not constant but increases steadily with time. If the particle thus comes to matter-dominate the universe much earlier than the canonical epoch (2.29), then it must subsequently decay (dominantly) into relativistic particles so that its energy density can be adequately redshifted, otherwise the bounds on the age and expansion rate of the universe today would be violated (Sato and Kobayashi 1977, Dicus *et al* 1978a). If such decays are into *interacting* particles such as photons or electromagnetically/strongly interacting particles which increase the entropy, the nucleon-to-photon ratio will decrease (Miyama and Sato 1978, Dicus *et al* 1978b).† As we have seen, the observationally inferred upper bound on the synthesized ^{4}He abundance implies an *upper* limit to η_{ns}, the nucleon-to-photon ratio during the nucleosynthesis epoch, while observations of luminous matter in the universe set a *lower* limit (3.31) to the same ratio today. Hence we can require particle decays after nucleosyntheis to not have decreased η by more than a factor $\eta_{\mathrm{ns}}/\eta_0$, having calculated the elemental yields (and η_{ns}) taking into account the increased expansion rate due to the decaying particle. However if the particle decays into *non-interacting* particles, e.g. neutrinos or hypothetical goldstone bosons which do not contribute to the entropy, then the only constraint comes from the increased expansion rate during nucleosynthesis. There may be additional effects in both cases if the decays create electron neutrinos/antineutrinos which can alter the chemical balance between neutrons and protons (Dicus *et al* 1978a) and thus affect the yields of D and ^{3}He (Scherrer 1984).

First we must calculate how the dynamics of the expansion are altered from the usual radiation-dominated case. Given the thermally-averaged self-annihilation cross-section of the x particle, one can obtain the relic abundance in ratio to photons the x particle would have at $T \ll m_{\mathrm{e}}$, assuming it is stable, using the methods outlined by Srednicki *et al* (1988) and Gondolo and Gelmini (1991). An approximate estimate may be obtained from the simple 'freeze-out' approximation (see Kolb and Turner 1990) of determining the temperature at which the self-annihilation rate falls behind the Hubble expansion rate. The surviving relic abundance is then given by the equilibrium abundance at this temperature:

$$\left(\frac{m_x}{\mathrm{GeV}}\right)\left(\frac{n_x}{n_\gamma}\right) \approx \left(\frac{\langle\sigma v\rangle}{8\times 10^{-18}\mathrm{GeV}^{-2}}\right)^{-1} . \tag{4.16}$$

Since we will generally be concerned with decay lifetimes much longer than $\approx 1\,\mathrm{sec}$, this can be taken to be the initial value of the decaying particle abundance. We can now identify the temperature T_{m} at which the particle energy density $\rho_x\,(\simeq m_x n_x)$ would

† This assumes thermalization of the released energy which is very efficient for decay lifetimes $\lesssim 10^5$ sec (Illarianov and Sunyaev 1975, Sarkar and Cooper 1984). For longer lifetimes thermalization is incomplete, but then the absence of a spectral distortion in the CMBR sets equally restrictive constraints on the decaying particle abundance (e.g. Ellis *et al* 1992, Hu and Silk 1994).

equal the radiation energy density $\rho_{\rm R}\,(=\pi^2 g_\rho/30T^4)$, viz.

$$T_{\rm m} \equiv \frac{60\,\zeta(3)}{g_\rho\,\pi^4}\frac{m_x n_x}{n_\gamma}\ . \tag{4.17}$$

If the particle decays at a temperature below $T_{\rm m}$, then it would have matter-dominated the universe before decaying and thus significantly speeded up the expansion. The usual time-temperature relationship (2.66) is thus altered to

$$t \simeq -\left[\frac{3M_{\rm P}^2}{8\pi(\rho_x+\rho_{\rm R})}\right]^{1/2}\int\frac{dT}{T} = \left(\frac{5}{\pi^3 g_\rho}\right)^{1/2}\frac{M_{\rm P}}{T_{\rm m}^2}\left[\left(\frac{T_{\rm m}}{T}-2\right)\left(\frac{T_{\rm m}}{T}+1\right)^{1/2}+2\right]. \tag{4.18}$$

This reduces in the appropriate limit ($T_{\rm m} \ll T$) to the radiation-dominated case. After the particles decay, the universe reverts to being radiation-dominated if the decay products are massless. If we assume that all the x particles decay *simultaneously* when the age of the universe equals the particle lifetime, then the temperature at decay, $T_{\rm d}$, is given by the above relationship setting $t = \tau_x$.

4.2.1. Entropy producing decays: First let us consider the case when the decays create electromagnetically interacting particles. Following Dicus *et al* (1978a,b) we assume that the effect of the decays is to cause a 'jump' in the temperature, which we obtain from energy conservation to be

$$T\,(t>\tau_x) = [T\,(t<\tau_x)]^{3/4}[T\,(t<\tau_x)+f_\gamma g_\rho T_{\rm m}]^{1/4}\ , \tag{4.19}$$

where f_γ is the fraction of ρ_x which is ultimately converted into photons. The resulting change in η is

$$\frac{\eta\,(t<\tau_x)}{\eta\,(t>\tau_x)} = \left(1+\frac{f_\gamma\,g_\rho\,T_{\rm m}}{2\,T_{\rm d}}\right)^{3/4} \leq \frac{\eta_{\rm ns}}{\eta_0}\ . \tag{4.20}$$

(In fact, radiative particle decays which follow the usual exponential decay law cannot *raise* the photon temperature in an adiabatically cooling universe (cf. Weinberg 1982), but only slow down the rate of decrease, as noted by Scherrer and Turner (1985). However their numerical calculation shows that this does not significantly affect the change in η, which turns out to be only $\approx 10\%$ larger than the estimate above.) From (4.18) we obtain $\tau_x \propto T_{\rm m}^{-1/2}T_{\rm d}^{3/2}$ for $T_{\rm m} \gg T_{\rm d}$, i.e. if the x particles decay well after the universe has become dominated by their energy density. In this approximation, the constraint on the decay lifetime is (Ellis *et al* 1985b)

$$\left(\frac{\tau_x}{\rm sec}\right) \lesssim 0.8\ f_\gamma^{3/2}\left(\frac{T_{\rm m}}{\rm MeV}\right)^{-2}\left(\frac{\eta_{\rm ns}}{\eta_0}\right)^2, \tag{4.21}$$

if we take $g_\rho = 3.36$, i.e. for $T_{\rm d} \ll m_{\rm e}$.

As mentioned above, the upper limit to $\eta_{\rm ns}$ corresponding to the conservative requirement $Y_{\rm p}(^4{\rm He}) < 0.25$ (3.57) depends on the extent to which the expansion rate during nucleosynthesis is influenced by the massive particle. Such limits were obtained by Kolb and Scherrer (1982) (following Dicus *et al* 1978b) who modified the standard BBN code to include a massive neutrino (with the appropriate energy density) and examined its effect on the elemental yields. The effect should be proportional to the neutrino energy density, which rises $\propto m_\nu$ as long as the neutrinos remain relativistic at decoupling, i.e. for m_ν less than a few MeV, and falls thereafter $\propto m_\nu^{-2}$ (e.g. Lee and Weinberg 1977). Indeed the synthesized abundances are seen to increase with increasing neutrino mass upto $m_\nu \approx 5\,\mathrm{MeV}$, and fall thereafter as m_ν increases further. Kolb and Scherrer found that a neutrino of mass $m_\nu \sim 0.1 - 10\,\mathrm{MeV}$ alters the ^{4}He abundance *more* than a massless neutrino and that the neutrino mass has to exceed 20 MeV before the change in the abundance becomes acceptably small, while for $m_\nu \gtrsim 25\,\mathrm{MeV}$ there is negligible effect on nucleosynthesis. (In fact the abundances of D and ^{3}He are also increased, and by a factor which may even exceed that for the ^{4}He abundance. This is because these abundances are sensitive to the expansion rate at $T \approx 0.04 - 0.08\,\mathrm{MeV}$ when the strong interactions which burn deuterium freeze-out, and the massive particle may come to matter-dominate the expansion precisely at this time.) From the results of Kolb and Scherrer (1982) it can be inferred that $\eta_{\rm ns} \propto T_{\rm m}^{-1/2}$ for $T_{\rm m} \gtrsim 10^{-2}\,\mathrm{MeV}$ and $\eta_{\rm ns} \approx$ constant for $T_{\rm m} \lesssim 10^{-2}\,\mathrm{MeV}$, hence it is easy to generalize the constraints obtained for neutrinos to any other particle which is non-relativistic during nucleosynthesis, i.e. with a mass larger than a few MeV. Ellis *et al* (1985b) used these values of $\eta_{\rm ns}$ to obtain the following restrictions on the energy density of the decaying particle as a function of the lifetime:

$$
\begin{aligned}
\left(\frac{m_x}{\mathrm{GeV}}\right)\left(\frac{n_x}{n_\gamma}\right) &\lesssim 6.0\times10^{-3}\left(\frac{\tau_x}{\mathrm{sec}}\right)^{-1/3} f_\gamma^{-1/2}\left(\frac{\eta_0}{1.8\times10^{-11}}\right)^{-2/3} \\
&\quad \text{for} \quad t_{\rm ns} \lesssim \tau_x \lesssim 3.8\times10^5 f_\gamma^{3/2}\mathrm{sec}\ , \\
&\lesssim 5.1\times10^{-2}\left(\frac{\tau_x}{\mathrm{sec}}\right)^{-1/2} f_\gamma^{3/4}\left(\frac{\eta_0}{1.8\times10^{-11}}\right)^{-1} \\
&\quad \text{for} \quad \tau_x \gtrsim 3.8\times10^5 f_\gamma^{3/2}\mathrm{sec}\ .
\end{aligned}
\tag{4.22}
$$

These constraints should be valid for radiative decays occuring after the begining of nucleosynthesis at $t_{\rm ns} \simeq 180\,\mathrm{sec}$ as indicated by the dotted line in figure 13(a). Scherrer and Turner (1988a) have performed a numerical study in which the cosmological evolution is computed taking into account the exponentially decreasing energy density of the massive particle and the correspondingly increasing energy density of its massless decay products, without making any approximations (c.f. the assumption above that $T_{\rm m} \gg T_{\rm d}$). These authors were thus able to study how the constraint weakens as τ_x decreases below $t_{\rm ns}$, as shown by the full line in figure 13(a). The reason the curve turns up sharply is that helium synthesis is unaffected by decays which occur prior to the epoch ($T \approx 0.25\,\mathrm{MeV}$) when the n/p ratio freezes out (see (3.24)).† In addition, Scherrer and Turner studied the effect on the D and ^{3}He abundances and imposed the indirect bound $[(\mathrm{D} + {}^3\mathrm{He})/\mathrm{H}]_\mathrm{p} \lesssim 10^{-4}$ (3.67) to obtain a more restrictive constraint shown as the dashed line in figure 13(a). All these curves are drawn assuming $f_\gamma = 1$ and can be scaled for other values of f_γ (or η_0) following (4.22).

When the decays occur *before* the nucleosynthesis era, the generation of entropy can only be constrained by requiring that the baryon asymmetry generated at earlier epochs should not have been excessively diluted, as was noted by Harvey *et al* (1981). Scherrer and Turner (1988a) assumed that the initial nucleon-to-photon ratio is limited by $\eta_\mathrm{i} < 10^{-4}$, as was believed to be true for GUT baryogenesis (see Kolb and Turner 1983), and combined it with the lower limit on the value of η today, to obtain the constraints shown as dot-dashed lines in figure 13(a). Obviously these are very model dependent since the initial value of η may be higher by several orders of magnitude, as is indeed the case in various non-GUT models of baryogenesis (see Dolgov 1992).

4.2.2. 'Invisible' decays: We should also consider the possibility that $f_\gamma = 0$, i.e. the decays occur into massless particles such as neutrinos or hypothetical goldstone bosons. In this case there is no change in the entropy, hence the constraints discussed above do not apply. However we can still require that the speed-up of the expansion rate during nucleosynthesis not increase the synthesized abundances excessively. As mentioned earlier, Kolb and Scherrer (1982) found that when a massive neutrino was incorporated into the standard BBN code, the observational bound $Y_\mathrm{p}(^4\mathrm{He}) < 0.25$ (3.57) is respected when the neutrino mass exceeds 20 MeV. We can generalize their result to any particle which is non-relativistic at nucleosynthesis by demanding that it should not matter-dominate the expansion any earlier than a 20 MeV neutrino. This implies the constraint (Ellis *et al* 1985b)

$$\left(\frac{m_x}{\mathrm{GeV}}\right)\left(\frac{n_x}{n_\gamma}\right) \lesssim 1.6 \times 10^{-4} , \tag{4.23}$$

which is valid for particles which decay after nucleosynthesis, i.e. for $\tau_x \gtrsim t_\mathrm{ns}$ as indicated by the dotted line in figure 13(b). Scherrer and Turner (1988b) obtain a similar requirement from a detailed numerical calculation, as shown by the full line in the same figure. Again, they were able to study how the constraint relaxes as τ_x becomes smaller than t_ns. Since the decay products are massless, the effect is then the same as the addition of new relativistic degrees of freedom. Imposing the additional indirect bound $[(\mathrm{D} + {}^3\mathrm{He})/\mathrm{H}]_\mathrm{p} \lesssim 10^{-4}$ (3.67), equivalent in this context to allowing one new

† We have corrected for the fact that these authors referred to the value of n_x/n_γ at $T \approx 100\,\mathrm{MeV}$, i.e. before $\mathrm{e}^+\mathrm{e}^-$ annihilation, while we always quote the value at $T \ll m_\mathrm{e}$, i.e. the abundance the particle would have today if it had not decayed. Also, they adopted a slightly different bound: $\eta_0 \gtrsim 3 \times 10^{-11}$.

neutrino species (see (4.5)), then yields the constraint

$$\left(\frac{m_x}{\mathrm{GeV}}\right)\left(\frac{n_x}{n_\gamma}\right) \lesssim 9.8 \times 10^{-4} \left(\frac{\tau_x}{\mathrm{sec}}\right)^{-1/2} , \tag{4.24}$$

valid for $\tau_x \ll \tau_{\mathrm{ns}}$ as shown by the dashed line in figure 13(b). This requirement is *more* restrictive than the corresponding one (4.22) for decays which create entropy, hence the two constraints should be weighted with the appropriate branching ratios in order to obtain the correct constraint for a particle whose decays produce both non-interacting particles and photons. Neutrino decay products actually present a special case since these are not entirely non-interacting. Indeed if decaying particles create a (non-thermal) population of electron (anti)neutrinos, these will bias the chemical balance between protons and neutrons towards the latter through the reaction $\mathrm{ne}^+ \to \mathrm{p}\bar{\nu}_{\mathrm{e}}$; the reverse reaction $\mathrm{n}\nu_{\mathrm{e}} \to \mathrm{pe}^-$ is negligible by comparison since protons always outnumber neutrons by a large factor (Scherrer 1984). This effect is important only when the mass of the decaying particle is of $\mathrm{O}(10)$ MeV as will be discussed later in the context of a massive unstable tau neutrino (see section 5.1).

Note also that if the particle lifetime exceeds the age of the universe then the only constraint comes from requiring that it respects the bound (2.28) on the present energy density. Using (4.16) and $\Omega_x h^2 = 3.9 \times 10^7 (m_x/\,\mathrm{GeV})(n_x/n_\gamma)$, this requires

$$\left(\frac{m_x}{\mathrm{GeV}}\right)\left(\frac{n_x}{n_\gamma}\right) \lesssim 2.6 \times 10^{-8} . \tag{4.25}$$

A particle which saturates this bound would of course be the (dominant) constituent of the dark matter; however from the preceeding discussion it is clear that such an abundance is still too small to have affected nucleosynthesis.

Far more stringent constraints than those discussed above come from consideration of the direct effects of the decay products on the synthesized elemental abundances. High energy photons or leptons from the decaying particles can initiate electromagnetic cascades in the radiation-dominated thermal plasma, thus creating many low energy photons with $E_\gamma \sim \mathrm{O}(10)$ MeV which are capable of photodissociating the light elements (Lindley 1979). Such photofissions can occur only for $t \gtrsim 10^4$ sec, i.e. after nucleosynthesis is *over*, since at earlier epochs the blackbody photons are energetic enough and numerous enough that photon-photon interactions are far more probable than photon-nucleus interactions (Lindley 1985). When the x particle decays into energetic quarks or gluons, these fragment into hadronic showers which interact with the ambient nucleons thus changing their relative abundances. (The alteration of elemental abundances by direct annihilation with antinucleons has also been considered (e.g Khlopov and Linde 1984, Ellis *et al* 1985b, Halm 1987, Dominguez-Tenreiro 1987); however Dimopoulos *et al* (1988) have shown that the effect of the hadronic showers is far more important.) If such hadronic decays occur *during* nucleosyntheis, the neutron-to-proton ratio is increased resulting in the production of more D and ^{4}He (Reno and Seckel 1988). However when hadronic decays occur *after* nucleosynthesis, the result is destruction of ^{4}He and creation of D and ^{3}He, as well as both ^{6}Li and ^{7}Li (Dimopoulos *et al* 1988).

When the x particle has both radiative and hadronic decay modes, the situation is then simplified by noting that for $\tau_x \sim 10^{-1} - 10^4\,$sec, radiative decays do not play a significant role while hadronic decays are constrained by the concommitant overproduction of D and ^{4}He by the hadronic showers (Reno and Seckel 1988). For longer lifetimes, the situation is more complicated since elements may be simultaneously both created and destroyed by photo- and hadro- processes. It has been argued that for $\tau_x \gtrsim 10^5\,$sec, the most stringent constraint on radiative decays comes from constraining the possible overproduction of D and ^{3}He through photofission of ^{4}He, since the simultaneous destruction of the former by photofission is negligible by comparison (Ellis *et al* 1985b, Juszkiewicz *et al* 1985). A somewhat weaker constraint obtains from constraining the depletion of the ^{4}He abundance itself (Ellis *et al* 1985b, Dimopoulos *et al* 1989). These constraints are strengthened if hadronic decays also occur since these too destroy ^{4}He and create D and ^{3}He. However all these constraints are found to be modified when the development of the electromagnetic cascades is studied taking $\gamma - \gamma$ (Möller) scattering into account; this reveals that ^{4}He destruction is significant only for $\tau_x \gtrsim 5 \times 10^6\,$sec (Ellis *et al* 1992). It has also been argued that in the interval $\tau_x \sim 10^3 - 10^5\,$sec, D is photodissociated but not ^{4}He, so that the strongest constraint on radiative decays now comes from requiring that D should not be excessively depleted (Juszkiewicz *et al* 1985, Dimopoulos *et al* 1989). Again, reexamination of the cascade process indicates that the appropriate interval is shifted to $\tau_x \sim 5 \times 10^4 - 2 \times 10^6\,$sec (Ellis *et al* 1992). This particular constraint may appear to be circumvented if hadronic decay channels are also open since hadronic showers *create* D; however such showers also create the rare isotopes ^{6}Li and ^{7}Li and are thus severely constrained by observational limits on their abundance (Dimopoulos *et al* 1989). This ensures that the D photofission constraint is not affected by such hadronic decays.

4.2.3. Electromagnetic showers: Let us begin by examining the manner in which a massive particle decaying into photons or leptons generates electromagnetic cascades in the radiation-dominated thermal plasma of the early universe. The dominant mode of energy loss of a high energy photon (of energy E_γ) is e^+e^- pair-production on the low energy blackbody photons (of energy ϵ_γ) while the produced electrons and positrons (of energy E_e) lose energy by inverse-Compton scattering the blackbody photons to high energies. Pair-production requires $E_\gamma \epsilon_\gamma \geq m_e^2$, while $E_e \epsilon_\gamma \geq m_e^2$ implies that scattering occurs in the Klein-Nishina regime in which the electron loses a large fraction of its energy to the scattered photon. Thus a primary photon or lepton triggers a cascade which develops until the photon energies have fallen below the pair-production threshold, $E_{\max} = m_{\rm e}^2/\epsilon_\gamma$. Subsequently the photons undergo Compton scattering on the electrons and pair-production on the ions of the thermal plasma. If the density of the blackbody photons is large enough, the cascade is termed 'saturated' implying that nearly all of the primary paricle energy is converted into photons with energy below $E_{\max}$. Note that such an electromagnetic cascade can be initiated even when the decay particle is a neutrino since it can initiate pair-production, $\nu\bar{\nu} \to {\rm e}^+{\rm e}^-$, off the (anti)neutrinos of the thermal background, if its energy is sufficiently high (e.g. Gondolo *et al* 1993), or even off the decay (anti)neutrinos (Frieman and Giudice 1989). Once high energy electrons have been thus created, the subsequent development of the shower proceeds as before.

The spectrum of the 'breakout' photons below the pair-production threshold was originally found by Monte Carlo simulations of the cascade process to be (Aharonian *et al* 1985, Dimopoulos *et al* 1988)

$$\begin{aligned} \frac{{\rm d}N}{{\rm d}E_\gamma} &\propto E_\gamma^{-3/2} \quad \text{for} \quad 0 \leq E_\gamma \leq E_{\max} \equiv \frac{m_{\rm e}^2}{\epsilon_\gamma}, \\ &\propto 0 \qquad \text{for} \quad E_\gamma > E_{\max} \ , \end{aligned} \tag{4.26}$$

when the background photons are assumed to be monoenergetic. Subsequently an analytic study of the kinetic equation for the cascade process showed that the spectrum actually steepens further to $E_\gamma^{-1.8}$ for $E_\gamma \gtrsim 0.3E_{\max}$ (Zdziarski and Svensson 1989). This feature had not been recognized in the Monte Carlo simulations due to insufficient statistics. In the cosmological context, the background photons are not monoenergetic but have a Planck distribution at temperature T. Naïvely we would expect that the pair-production threshold is then $E_{\max} \approx m_{\rm e}^2/T$. However the primæval plasma is radiation-dominated, i.e. the number density of photons is very large compared to the number density of electrons and nuclei. Hence even when the temperature is too low for a high energy photon to pair-produce on the *bulk* of the blackbody photons, pair-production may nevertheless occur on the energetic photons in the Wien tail of the Planck distribution (Lindley 1985). Although the spectrum here is falling exponentially with energy, the number of photons with $\epsilon_\gamma \gtrsim 25T$ is still comparable to the number of thermal electrons since $n_\gamma/n_{\rm e} \gtrsim 10^9$. Therefore pair-production on such photons is as important as Compton scattering on electrons or pair-production on ions, the respective cross-sections being all comparable. Hence the value of $E_{\max}$ is significantly lowered below the above estimate, as seen by equating the mean free paths against pair-production on photons and Compton scattering on electrons (Zdziarski and Svensson 1989):

$$E_{\max} \simeq \frac{m_{\rm e}^2}{20.4T\ [1 + 0.5\ \ln(\eta/7\times10^{-10})^2 + 0.5\ \ln(E_{\max}/m_{\rm e})^2]}\ . \tag{4.27}$$

(Note that at the energies relevant to photofission processes ($E_\gamma < 100\,{\rm MeV}$), pair-production on ions is unimportant by comparison with Compton scattering.) Although various authors have noted this effect, they have used quite different estimates of $E_{\rm max}$, viz. $m_{\rm e}^2/12T$ (Lindley 1985, Juszkiewicz *et al* 1985), $2m_{\rm e}^2/25T$ (Salati *et al* 1987), $m_{\rm e}^2/18T$ (Kawasaki and Sato 1987), $m_{\rm e}^2/25T$ (Dimopoulos *et al* 1988, 1989) and $m_{\rm e}^2/32T$ (Dominguez-Tenreiro 1987). Moreover all these authors assumed the spectrum to be of the form (4.26) whereas for a blackbody target photon distribution it actually steepens to $E_\gamma^{-1.8}$ for $E_\gamma \gtrsim 0.03 E_{\rm max}$ (Zdziarski 1988).

Subsequently it was noted that $\gamma-\gamma$ elastic scattering is the *dominant* process in a radiation-dominated plasma for photons just below the pair-production threshold (Zdziarski and Svensson 1989), hence $E_{\rm max}$ really corresponds to the energy for which the mean free paths against $\gamma-\gamma$ scattering and $\gamma-\gamma$ pair-production are equal. For a Planck distribution of background photons this is (Ellis *et al* 1992)

$$E_{\rm max} \simeq \frac{m_{\rm e}^2}{22T} \; ; \tag{4.28}$$

photons pair-produce above this energy and scatter elastically below it. Another effect of $\gamma-\gamma$ scattering is reprocessing of the cascade spectrum leading to further reduction in the number of high energy photons. The spectrum now falls like $E_\gamma^{-1.5}$ upto the energy $E_{\rm crit}$ where $\gamma-\gamma$ scattering and Compton scattering are equally probable and then steepens to E_γ^{-5} before being cutoff at $E_{\rm max}$ by the onset of pair-production (Zdziarski 1988). The value of $E_{\rm crit}$ depends weakly on the photon energy; at the energies of $\sim 2.5-25\,{\rm MeV}$ relevant to the photofission of light nuclei, it is

$$E_{\rm crit} \simeq \left(\frac{m_{\rm e}^2}{44T}\right)\left(\frac{\eta}{7\times10^{-10}}\right)^{1/3} , \tag{4.29}$$

i.e. effectively $E_{\rm crit}{\simeq}E_{\rm max}/2$.

We can now study how the yields in the standard BBN model are altered due to photofission by the cascade photons from a hypothetical decaying particle. Let ${\rm d}N_x/{\rm d}E$ denote the spectrum of high energy photons (or electrons) from massive particle decay, normalized as

$$\int_0^\infty E\frac{{\rm d}N_x}{{\rm d}E}{\rm d}E = f_\gamma m_x \; , \tag{4.30}$$

where f_γ is the fraction of the x particle mass released in the form of electromagnetically interacting particles (easily calculable once the decay modes and branching ratios are specified). A decay photon (or electron) of energy E initiates a cascade with the spectrum

$$\begin{aligned}\frac{{\rm d}n_E}{{\rm d}E_\gamma} &= \frac{24\sqrt{2}}{55}\frac{E}{E_{\rm max}^{1/2}}E_\gamma^{-3/2} \quad \text{for} \quad 0\le E_\gamma\le E_{\rm max}/2,\\ &= \frac{3}{55}EE_{\rm max}^3E_\gamma^{-5} \quad \text{for} \quad E_{\rm max}/2\le E_\gamma\le E_{\rm max},\\ &= 0 \quad \text{for} \quad E_\gamma > E_{\rm max},\end{aligned} \tag{4.31}$$

where we have normalized the cascade spectrum as

$$\int_0^{E_{\rm max}} E_\gamma \frac{{\rm d}n_E}{{\rm d}E_\gamma}{\rm d}E_\gamma = E\ , \qquad E_{\rm max} = \frac{m_{\rm e}^2}{22T}\ . \tag{4.32}$$

Recently, Kawasaki and Moroi (1995a,b) have claimed that numerical solution of the governing Boltzman equations yields a different cascade spectrum which has significant power beyond the cutoff $E_{\rm max}$ and is also less steep below $E_{\rm max}/2$. We note that Protheroe *et al* (1995) obtain results in agreement with those above from a detailed Monte Carlo simulation of the cascade process.

To write the balance equation for the change in the abundance of element i with total photofission cross-section σ_i (above threshold Q_i), we note that recombination of the dissociated nuclei, in particular D, is negligible for $t \gtrsim 10^4$ sec, hence (Ellis *et al* 1985b)

$$\frac{{\rm d}X_{\rm i}}{{\rm d}t}|_{\rm photo} = -\frac{{\rm d}n_x}{{\rm d}t}\int_0^\infty \frac{{\rm d}N_x}{{\rm d}E}{\rm d}E\left(\int_{Q_i}^E \frac{{\rm d}n_E}{{\rm d}E_\gamma}\frac{X_{\rm i}\sigma_i}{n_{\rm e}\sigma_{\rm C}}{\rm d}E_\gamma - \sum_{j\neq i}\int_{Q_i}^E \frac{{\rm d}n_E}{{\rm d}E_\gamma}\frac{X_j\sigma_{j\to i}}{n_{\rm e}\sigma_{\rm C}}{\rm d}E_\gamma\right), \tag{4.33}$$

where $\sigma_{j\to i}$ is the partial cross-section for photofission of element i to element j and $\sigma_{\rm C}$ is the cross-section for Compton scattering on the thermal electrons of density

$$n_{\rm e} \simeq \left(1 - \frac{Y}{2}\right)\ n_{\rm N} = \frac{7}{8}\eta n_\gamma\ , \tag{4.34}$$

for a H+He plasma, taking $Y(^4{\rm He}) = 0.25$. (Anticipating the stringent constraints on the particle abundance to be derived shortly, we assume that η is not altered significantly by the entropy released in particle decays.) Since the number density of x particles decreases from its initial value $n_x^{\rm i}$ as

$$\frac{{\rm d}n_x}{{\rm d}t} = -\frac{n_x^{\rm i}}{\tau_x}\exp\left(-\frac{t}{\tau_x}\right), \tag{4.35}$$

the time-integrated change in the elemental abundance (in a comoving volume) is given by (Ellis *et al* 1992)

$$\begin{aligned}
\int_{t_i^{\rm min}}^\infty \frac{{\rm d}X_{\rm i}}{{\rm d}t}|_{\rm photo}\ {\rm d}t &\simeq \left(m_x\frac{n_x}{n_\gamma}\right)\frac{f_\gamma}{\eta}\left(1-\frac{Y}{2}\right)^{-1}\left[-X_{\rm i}\beta_i(\tau_x) + \sum_{j\neq i}X_j\beta_{j\to i}(\tau_x)\right], \\
\beta_i(\tau_x) &\equiv \int_{t_i^{\rm min}}^\infty \frac{{\rm d}t}{\tau_x}\exp\left(-\frac{t}{\tau_x}\right)\int_{Q_i}^{E_{\rm max}(t)}\left(\frac{1}{E}\frac{{\rm d}n_E}{{\rm d}E_\gamma}\right)\frac{\sigma_i(E_\gamma)}{\sigma_{\rm C}(E_\gamma)}{\rm d}E_\gamma\ , \\
\beta_{j\to i}(\tau_x) &\equiv \int_{t_j^{\rm min}}^\infty \frac{{\rm d}t}{\tau_x}\exp\left(-\frac{t}{\tau_x}\right)\int_{Q_j}^{E_{\rm max}(t)}\left(\frac{1}{E}\frac{{\rm d}n_E}{{\rm d}E_\gamma}\right)\frac{\sigma_{j\to i}(E_\gamma)}{\sigma_{\rm C}(E_\gamma)}{\rm d}E_\gamma\ ,
\end{aligned} \tag{4.36}$$

(We have dropped the superscript i on n_x above and hereafter since, as before, we will be concerned with particles which decay late, long after they fall out of chemical equilibrium. Hence the usual freeze-out abundance (e.g. (4.16)), can be sensibly taken to be the initial abundance, with due allowance made for whether the particle decays occur before or after e^+e^- annihilation.) The time $t_i^{\min}$ at which photofission of element i starts can be computed (from the time-temperature relationship for a radiation-dominated universe (2.66) with $g_\rho = 3.36$ for $T \ll m_e$) corresponding to the critical temperature at which the cascade cutoff energy $E_{\max}$ (4.28) equals the threshold Q_i for the most important photofission reactions:

$$\begin{aligned} &Q_{\gamma D \to pn} = 2.23\ \text{MeV}, \\ &Q_{\gamma ^3\text{He} \to pD} = 5.49\ \text{MeV}, \quad Q_{\gamma ^3\text{He} \to 2\,pn} = 7.72\ \text{MeV}, \\ &Q_{\gamma ^4\text{He} \to pT} = 19.8\ \text{MeV}, \quad Q_{\gamma ^4\text{He} \to n^3\text{He}} = 20.6\ \text{MeV}, \quad Q_{\gamma ^4\text{He} \to pnD} = 26.1\ \text{MeV}, \\ &\vdots \end{aligned} \tag{4.37}$$

As we shall see, the decaying particle abundance is constrained to be sufficiently small that the effect on the dynamics of the expansion is negligible. Hence it is consistent to take the input abundances to be those obtained in the standard BBN model and study how these may be altered by photofission processes.

The integrals β_i and $\beta_{j \to i}$ have been computed numerically for various values of τ_x using the cascade spectrum (4.31) and the known cross-sections for photofission processes (see Faul *et al* 1981, Gari and Hebach 1981, Govaerts *et al* 1981). As seen in figure 14, β_i rises sharply from zero above a critical value of τ_x (which increases as the square of the photofission threshold Q_i), peaks at a value which is nearly the same ($\approx 1\,\text{GeV}^{-1}$) for all light elements, and subsequently falls off rather slowly with increasing τ_x. This reflects the fact that the relevant photofission cross-sections are all of order a few millibarns above threshold and fall rapidly thereafter with increasing energy. Photofission begins when the cascade cutoff energy just crosses the photofission threshold and the dominant effect is that of photons with energies just over this threshold. This implies that when photofission of ^{4}He occurs, the resultant production of D and ^{3}He ($\gamma ^4\text{He} \to p\text{T}, n^3\text{He}, pn\text{D};\ \text{T} \to {}^3\text{He}\, e^- \bar{\nu}_e$) far dominates their destruction since the abundance of ^{4}He is $\sim 10^4$ times greater (Ellis *et al* 1985b). As seen in figure 14, photofission of D begins at $\tau_x \approx 10^4$ sec and becomes significant at $\tau_x \approx 10^5$ sec while photofission of ^{4}He begins at $\tau_x \approx 10^6$ sec and begins significant at $\tau_x \approx 10^7$ sec. Therefore for $\tau_x \gtrsim 10^6$ sec, (4.36) can be simplified to read, for the difference between the initial and final mass fractions of D + ^{3}He:

$$X_f(\text{D} + {}^3\text{He}) - X_i(\text{D} + {}^3\text{He}) \simeq Y_i({}^4\text{He}) \left(m_x \frac{n_x}{n_\gamma} \right) \frac{f_\gamma}{\eta} \left(1 - \frac{Y}{2} \right)^{-1} r\ \beta_{^4\text{He}}(\tau_x), \tag{4.38}$$

where, $r \equiv [(\frac{3}{4}\sigma_{\gamma ^4\text{He} \to n^3\text{He},\ p\text{T}} + \frac{1}{2}\sigma_{\gamma ^4\text{He} \to pn\text{D}})/\sigma_{\gamma ^4\text{He} \to \text{all}}] \simeq 0.5$, and the subscripts i and f refer to the initial and final values. To obtain the most conservative constraint on the abundance of the decaying particle we must consider the maximum value allowed for $[X_f(\text{D} + {}^3\text{He}) - X_i(\text{D} + {}^3\text{He})]\eta / Y_i({}^4\text{He})$. Since ^{4}He cannot have been destroyed significantly (without overproducing D and ^{3}He) we take its initial abundance to be the maximum permitted value, i.e. $Y_{\rm i}(^4{\rm He}) < 0.25$, which implies that $\eta < 9.2 \times 10^{-10}$ (3.75). Hence a minimum mass fraction $X_{\rm i}({\rm D} + {}^3{\rm He}) > 3.8 \times 10^{-5}$ would have been primordially synthesized (see figure 5). The maximum final abundance after photoproduction consistent with 'standard' galactic chemical evolution is bounded by $X_{\rm f}({\rm D}+{}^3{\rm He}) \lesssim 2.5\times10^{-4}$, using (3.67) and taking into account that comparable numbers of ^{3}He and D nuclei are photoproduced. Using these numbers and taking $f_\gamma = 1$ yields the upper limit (full line) on $m_x n_x/n_\gamma$ shown in figure 15 above which D + ^{3}He is overproduced. For reference, the dashed line indicates the constraint obtained earlier by Ellis *et al* (1985b) using the same argument but with a less sophisticated treatment of the cascade process. A similar constraint was obtained by Juszkiewicz *et al* (1985). Recently, Protheroe *et al* (1995) have performed a Monte Carlo simulation of the cascade process and quoted bounds on $\Omega_x/\Omega_{\rm N}$ for three choices of $(\eta/10^{-10}) = 2.7, 3.3, 5.4$. We have rescaled their bound taking η to be 9.2×10^{-10} for fair comparison with Ellis *et al* (1992) and plotted this as the dotted line in figure 15. The two results are seen to be in good agreement. We cannot however reproduce either the *less* stringent constraint quoted by Kawasaki and Sato (1987) or the *more* stringent constraint given by Kawasaki and Moroi (1995a);† since both these results were obtained entirely by numerical integration of the governing equations, we cannot easily identify the reasons for the discrepancy. Dimopoulos *et al* (1989) did not consider the constraint on the decaying particle abundance from photoproduction of D + ^{3}He. These authors criticized Ellis *et al* (1985b) for having neglected the photofission of D by comparison, but as shown above this is quite justified since the correction is only of $O(10^{-4})$.

For $\tau \lesssim 10^6$ sec, photofission of ^{4}He is not significant so D and ^{3}He are not produced but only destroyed. Assuming that hadronic decay channels are not open, (4.33) now reads for the change in the D mass fraction alone

$$\frac{X_{\rm i}({\rm D})}{X_{\rm f}({\rm D})} \simeq \exp\left[\left(\frac{m_x n_x}{n_\gamma}\right)\frac{f_\gamma}{\eta}\left(1-\frac{Y}{2}\right)^{-1}\beta_{\rm D}(\tau_x)\right] \tag{4.39}$$

Again, to obtain the most conservative constraint on the particle abundance, we must maximize the quantity $\eta \ln[X_{\rm i}({\rm D})/X_{\rm f}({\rm D})]$ subject to the observational constraint that the D abundance after photofission must exceed the observational bound (3.59). Using (3.51) we see that this quantity peaks at $\approx 6.5 \times 10^{-10}$ for $\eta \approx 4 \times 10^{-10}$. The corresponding upper limit on the decaying particle abundance is indicated in figure 15 above which D is excessively depleted. The dot-dashed line alongside is the upper limit obtained by Dimopoulos *et al* (1989) from similar considerations but ignoring

† Kawasaki and Moroi (1995b) claim that the discrepancy arises because Ellis *et al* (1992) and Protheroe *et al* (1995) did not allow for the standard synthesis of D + ^{3}He. In fact these authors did so, albeit in a more conservative (and consistent) manner.

$\gamma - \gamma$ scattering. All the above constraints apply to any decaying particle which can generate electromagnetic cascades above the photofission threholds; this requires $m_x \approx 2E_\gamma \gtrsim 5 - 50\,\text{MeV}$ depending on which element is being considered (see (4.37)).

4.2.4. Hadronic showers: As mentioned earlier, when hadronic decay channels are open, D is *produced* by hadronic showers and this requires reconsideration of the constraint derived above. In fact, even if the particle decays exclusively into photons, the resulting electromagnetic cascades will be effectively hadronic for $E_\gamma \epsilon_\gamma > \text{O}(1)\text{GeV}^2$; furthermore there is always a $\approx 1\%$ probability for the (virtual) decay photon to convert into a $q\bar{q}$ pair over threshold. Hence hadronic showers will be generated if the particle is heavier than $\approx 1\,\text{GeV}$ even if it has no specific hadronic decay channels (Reno and Seckel 88). As discussed in detail by Dimopoulos *et al* (1988, 1989), the main effect of hadronic showers is the destruction of the ambient ^{4}He nuclei and the creation of D, ^{3}He, ^{6}Li and ^{7}Li. The average number of i nuclei created per x particle decay, ξ_i, can be computed by modelling the hadronic shower development using e^+e^- jet data. The balance equation for an elemental abundance now reads

$$\frac{\mathrm{d}X_i}{\mathrm{d}t} = \frac{\mathrm{d}X_i}{\mathrm{d}t}|_{\text{photo}} + \frac{\mathrm{d}X_i}{\mathrm{d}t}|_{\text{hadro}} , \tag{4.40}$$

where the first term on the RHS is given by (4.33) and the second term is (Dimopoulos *et al* 1988, 1989)

$$\frac{\mathrm{d}X_i}{\mathrm{d}t}|_{\text{hadro}} = r_{\text{B}}^{\star}\xi_i\frac{\mathrm{d}n_x}{\mathrm{d}t} , \qquad r_{\text{B}}^{\star} \equiv \left(\frac{\nu_{\text{B}}}{5}\right) r_{\text{B}}\mathcal{F} . \tag{4.41}$$

Here $r_{\text{B}}^{\star}$ is an 'effective' baryonic branching ratio defined in terms of the true baryonic branching ratio r_{B}, the baryonic multiplicity ν_{B}, and a factor $\mathcal{F}$ representing the dependence of the yields ξ_i on the energy of the primary shower baryons. The e^+e^- jet production data suggests that for $m_x = 1\,\text{TeV}$, there are ≈ 5 nucleon-antinucleon pairs produced with $\approx 5\,\text{GeV}$ energy/nucleon. For other values of m_x, ν_{B} depends logarithmically on the energy, except near the baryon production threshold where the dependence is somewhat stronger.

Considering the effects of hadroproduction alone, (4.41) integrates to read (Dimopoulos *et al* 1989)

$$\frac{n_x^{\text{i}}}{n_\gamma} < \frac{(\mathcal{N}_i^{\text{max}} - \mathcal{N}_i^{\text{min}})\,\eta}{r_{\text{B}}^{\star}\,\xi_i} , \tag{4.42}$$

where $\mathcal{N}_i^{\text{max}}$ and $\mathcal{N}_i^{\text{min}}$ are, respectively, the maximum (observed) and minimum (synthesized) abundance of element i by number, relative to hydrogen. This constraint can be imposed on the hadroproduction of D, ^{3}He, ^{6}Li and ^{7}Li. Taking $\mathcal{N}_{^7\text{Li}}^{\text{max}} \approx 2\times10^{-10}$ and $\mathcal{N}_{^7\text{Li}}^{\text{min}} \approx 5\times10^{-11}$, this gives (Dimopoulos *et al* 1989)

$$\frac{n_x^{\text{i}}}{n_\gamma} < 1.5\times10^{-5}\frac{\eta}{r_{\text{B}}^{\star}} . \tag{4.43}$$

A similar constraint follows from requiring $\mathcal{N}^{\max}_{(\mathrm{D}+{}^3\mathrm{He})} \approx 10^{-4}$. An even stricter constraint can be obtained if we assume that the primordial abundance of ^{6}Li did not exceed its presently observed value, i.e. $\mathcal{N}^{\max}_{^6\mathrm{Li}} \approx 10^{-11}$. This yields (Dimopoulos *et al* 1989)

$$\frac{n_x^{\mathrm{i}}}{n_\gamma} < 3 \times 10^{-7} \frac{\eta}{r_{\mathrm{B}}^{\star}} \ . \tag{4.44}$$

The fact that these constraints are so very restrictive considerably simplifies the situation when *both* electromagnetic and hadronic showers occur. For a given x particle abundance, the hadronic branching ratio must be very small in order not to overproduce lithium. This ensures that the production of D by hadronic showers is quite negligible relative to its production by electromagnetic showers. This is borne out by numerical solution of (4.40) taking both kinds of showers into account (Dimopoulos *et al* 1989). It is true that in a small region of parameter space ($\tau_x \sim 10^5 - 10^6$ sec, $m_x n_x/n_{\mathrm{N}} \sim 10^1 - 10^3$ GeV, $r_{\mathrm{B}}^{\star} n_x/n_{\mathrm{N}} \sim 10^{-4} - 10^{-2}$), the photodestruction of D *is* compensated for by hadroproduction of D but this also results in the production of an excessive amounts of ^{6}Li. If this is indeed inconsistent with observations (e.g. Steigman *et al* 1993), the constraints derived from consideration of photofission processes are *not* evaded even if hadronic decay channels are also open.

For $\tau_x < 10^4$ sec, photofission does not occur for any element and standard nucleosynthesis is unaffected by electromagnetic showers. However hadronic showers can induce interconversions between the ambient protons and neutrons thus changing the equilibrium n/p ratio. This has been studied in detail by Reno and Seckel (1988) as discussed below. The transition rate for a thermal nucleon to convert to another nucleon is the usual weak interaction rate plus the rate due to hadronic showers, given by

$$\Gamma_{\mathrm{p}\to\mathrm{n}} = \frac{\Gamma_x n_x}{X(\mathrm{p})\, n_{\mathrm{N}}} \sum \mathcal{P}_{x_i} f^i_{\mathrm{pn}} \ , \qquad \Gamma_{\mathrm{n}\to\mathrm{p}} = \frac{\Gamma_x n_x}{X(\mathrm{n})\, n_{\mathrm{N}}} \sum \mathcal{P}_{x_i} f^i_{\mathrm{np}} \ , \tag{4.45}$$

where, $\Gamma_x \equiv \tau_x^{-1}$, $X(\mathrm{p})$ and $X(\mathrm{n})$ are the proton and neutron fractions, $\mathcal{P}_{x_i}$ is the average number of hadronic species i per x particle decay and f^i_{pn}, f^i_{np} are the probabilities for i to induce the respective transitions. The fragmentation process can be modelled using data on jet multiplicities from e^+e^- annihilation experiments (Reno and Seckel 1988):

$$\mathcal{P}_{x_i} \simeq N_{\mathrm{jet}} \langle n_{\mathrm{ch}}(E_{\mathrm{jet}})\rangle\, B_{\mathrm{h}} \left(\frac{n_i}{n_{\mathrm{ch}}}\right) . \tag{4.46}$$

Here B_{h} is the hadronic branching ratio for x decay, N_{jet} is the number of jets, and n_i, the charge multiplicity of species i, has been expressed as a fraction of the average charge multiplicity $\langle n_{\mathrm{ch}}(E_{\mathrm{jet}})\rangle$ at a given energy E_{jet}. The transition probability is computed as the ratio of the strong interaction rate to the sum of the decay and absorption rates for the injected hadrons:

$$f^i_{\mathrm{pn}} = \frac{\Gamma^i_{\mathrm{pn}}}{\Gamma^i_D + \Gamma^i_A} \ , \qquad f^i_{\mathrm{np}} = \frac{\Gamma^i_{\mathrm{np}}}{\Gamma^i_D + \Gamma^i_A} \ . \tag{4.47}$$

When the decaying particle carries no baryon number, the decay hadrons can be thought of as being injected in pairs so that i can refer to mesons as well as to baryon-antibaryon pairs (i.e. $i = n\bar{n}, p\bar{p}, \ldots$). The injected hadrons (except K_L) are stopped before they interact with the ambient neutrons so that threhold values of cross-sections can be used (Reno and Seckel 1988). The variable quantifying the effect of hadronic decays is then the x particle abundance multiplied by a parameter F defined as

$$F \equiv \frac{N_{\rm jet} B_{\rm h}}{2} \frac{\langle n(E_{\rm jet})\rangle}{\langle n(E_{33\rm GeV})\rangle} \ , \tag{4.48}$$

so that $F \simeq 1$ for $m_x = 100\,\text{GeV}$, if we take $E_{\rm jet} \simeq m_x/3$, $n_{\rm jet} B_{\rm h} = 2$, i.e. assuming that x decays into 3 particles at the parton level and $N_{\rm jet}$ equals the number of (non-spectator) quarks at the parton level.

The neutron fraction in the thermal plasma is always less than 0.5, being $X_{\rm n} \simeq 0.2$ at $T \simeq 1\,\text{MeV}$ where the weak interaction rate freezes-out (3.13), and decreasing by beta decay to 0.12 at 0.09 MeV (3.35) when nuclear reactions begin. Since there are always more protons than neutrons, the overall effect of hadronic decays in the interval $\tau_x \sim 1{-}200\,\text{sec}$ is to convert protons into neutrons. (For injection of $p\bar{p}$ pairs, the neutron fraction is actually reduced but this is compensated for by the effects of mesons and $n\bar{n}$ injection.) The additional neutrons thus produced are all synthesized into ^{4}He and hence hadronic decays in this lifetime interval are constrained by the observational upper bound to the helium abundance. Reno and Seckel (1988) obtain upper limits on $F n_x/n_\gamma$ as a function of τ_x for $\eta = 3 \times 10^{-10}$ and 10^{-9}, adopting the bound $Y_{\rm p}(^4\text{He}) < 0.26$. We have rescaled their results (for the case when x does not itself carry baryon number) to the more stringent constraint $Y_{\rm p}(^4\text{He}) < 0.25$ (3.57); this requires that we restrict ourselves to the case $\eta = 3 \times 10^{-10}$ since for $\eta = 10^{-9}$, $Y_{\rm p}$ already exceeds 0.25 even in the absence of x decays (see also Lazarides *et al* 1990). We calculate the resulting upper limit on $m_x n_x/n_\gamma$ taking $B_{\rm h} = 1$ (the limit scales inversely as $B_{\rm h}$) and show this in figure 15. This constraint gets more stringent as τ_x increases from 0.1 sec to 100 sec since the neutron fraction is dropping in this time interval. At later times the neutron fraction is effectively zero (since all neutrons are now bound in nuclei) and the only free neutrons are those created by x decay. These can bind into D but D cannot burn further to ^{4}He since the corresponding reaction rate is now too low due to the small densities. For $\tau_x \sim 100 - 1000\,\text{sec}$, the ^{3}He abundance is also increased by $\text{D} - \text{D}$ burning. For $\tau_x \gtrsim 10^4\,\text{sec}$ the released neutrons decay before forming D. Hence in the interval $10^2 - 10^4$ sec, the appropriate constraint on hadronic decays is the indirect bound $(\text{D} + {^3\text{He}})/\text{H} < 10^{-4}$ (3.67). The corresponding upper limit on $m_x n_x/n_\gamma$, extracted from Reno and Seckel (1988), is also shown in figure 15.

The above arguments apply to *any* form of energy release in the early universe, for example to annihilations of massive particles as they turn non-relativistic and freeze-out. This has been considered by Hagelin and Parker (1990) and by Frieman *et al* (1990), however their modelling of the cascade process was not accurate (e.g. $\gamma - \gamma$ scattering was not included). To obtain the correct constraints, the bounds shown in figures 13 and 15 should be imposed on the energy released in such annihilations.

The constraints derived from considerations of entropy generation (section 4.2.1) and speedup of the expansion rate (section 4.2.2) apply to any particle which is non-relativistic during nucleosynthesis, i.e. heavier than $\sim 0.1\,\mathrm{MeV}$, and are independent of the mass (except insofar as the mass may determine the relic abundance). However the bounds based on the development of electromagnetic (section 4.2.3) and hadronic (section 4.2.4) cascades require the mass of the decaying (or annihilating) particle to be significantly higher. For example, to generate an electromagnetic shower capable of efficiently photodissociating D, the initiating photon/electron must have an energy exceeding twice the relevant threshold (4.37), i.e. about 5 MeV and this would require the decaying particle to be at least 10 MeV in mass; for ^{4}He, the required mass is closer to 100 MeV. To generate a hadronic cascade, the mass would have to be even higher, typically in excess of a few hundred MeV. Thus when dealing with a mass of O(MeV), e.g. a massive ν_τ, it is more reliable to just calculate the spectrum of the photons scattered by the decay $e^\pm$ and then evaluate the extent to which say deuterium is photodissociated (Sarkar and Cooper 1984, Scherrer 1984).†

5. Applications

So far we have attempted to keep the discussion of constraints as 'model-independent' as possible in order that the results may be applied to any type of particle, including those which have not yet been thought of! Of course most discussions in the literature refer to specific particles, whether known or hypothetical. Of the known particles, the most interesting are neutrinos since laboratory experiments have set only weak limits on their properties. Turning to hypothetical particles, the most interesting are those predicted by suggested solutions to the naturalness problems of the Standard Model, e.g. technicolour and supersymmetry. Constraints on both categories have important implications for the nature of the dark matter in the universe.

† The inverse-Compton energy loss rate $\langle\dot{E}\rangle$ is high in a radiation-dominated universe, so that $-\int \mathrm{d}\,E'/\langle\dot{E}'\rangle \ll t$ at the epochs of interest. Hence the relevant transport equation for the electrons (see Blumenthal and Gould 1970) can be simply solved even in the relativistic Klein-Nishina limit to obtain the electron energy spectrum (modified by inverse-Compton scattering) given the source spectrum from neutrino decays. The spectrum of the Compton scattered high energy photons is then obtained by appropriate integration over the blackbody source distribution.

5.1. Neutrinos

The oldest and most popular application of cosmological constraints has been to neutrinos. Although neutrinos are massless in the Standard Model and interact weakly, their large relic abundance ensures that even a small neutrino mass or magnetic moment would have observable consequences in cosmology. These properties arise in (unified) theories incorporating new physics, e.g. lepton number violation at high energies, hence the cosmological arguments provide a sensitive probe of such physics. We discuss below only those constraints which arise from nucleosynthesis; other cosmological constraints, e.g. from stellar evolution, have been discussed in a number of recent reviews (see Kolb *et al* 1989, Fukugita and Yanagida 1994, Gelmini and Roulet 1995).

5.1.1. Neutrino masses: Combining the relic neutrino abundance (2.69) with the observational bound (2.28) on the present energy density imposed by the age and expansion rate of the universe, one obtains the well-known upper limit (Gershteĭn and Zeldovich 1966, Cowsik and McCleland 1972, Szalay and Marx 1976)

$$\sum_i m_{\nu_i} \left(\frac{g_{\nu_i}}{2}\right) \lesssim 94 \ \text{eV} \ , \tag{5.1}$$

where the sum is over all species which were relativistic at decoupling, i.e. with $m_{\nu_i} \lesssim 1\,\text{MeV}$. Alternatively, if the neutrino is more massive and falls out of chemical equilibrium before kinetic decoupling with the relic abundance (4.16), then one obtains the *lower* limit (Lee and Weinberg 1977, Dicus *et al* 1977, Vysotskiĭ *et al* 1977)

$$m_{\nu_i} \gtrsim 2\,\text{GeV} \ . \tag{5.2}$$

Thus no stable neutrino with Standard Model weak interactions can have a mass in the range $\sim 100\,\text{eV} - 2\,\text{GeV}$. The experimental mass limits are (Particle Data Group 1996)

$$\text{“}m_{\nu_e}\text{''} < 5.1 \ \text{eV}, \quad \text{“}m_{\nu_\mu}\text{''} < 160 \ \text{keV}, \quad \text{“}m_{\nu_\tau}\text{''} < 24 \ \text{MeV}, \tag{5.3}$$

all at 95% c.l.. (The quotes are to remind us that these are really bounds on the mass eigenstates coupled dominantly to the respective charged leptons (Schrock 1981) as discussed below.) Thus only the electron neutrino mass is experimentally known to be below the cosmological upper bound. The muon and tau neutrinos are then required by cosmology to also be lighter than 100 eV, or have their relic abundance suppressed by some means, e.g. decays or enhanced self-annihilations.

The Standard Model contains only massless neutrinos in left-handed (LH) doublets but its successful phenomenology would be unaffected by the addition of right-handed (RH) neutrinos as isosinglets and/or additional Higgs bosons to violate lepton number conservation, thus allowing a Dirac and/or Majorana mass (see Langacker 1988, Mohapatra and Pal 1991, Valle 1991, Gelmini and Roulet 1995). A Majorana mass term is naturally generated by a dimension-5 operator in extensions of the SM with intrinsic left-right symmetry and lepton-quark symmetry, such as $SO(10)$ and its subgroup $SU(2)_{\rm L}\otimes SU(2)_{\rm R}\otimes U(1)_{B-L}$. A Dirac neutrino can be viewed as a mass-degenerate pair of Majorana neutrinos with opposite CP eigenvalues (see Kayser *et al* 1989) and arises, for example, in extended $SO(10)$ models. In general, one should consider a mass matrix mixing p two-component neutrino spinors belonging to $SU(2)_{\rm L}$ doublets with q two-component neutrino spinors which are singlets under $SU(2)_{\rm L}\otimes U(1)_Y$. The *LEP* measurement of the Z^0 decay width fixes p to be 3 but allows any number of singlets (more precisely, any number of light neutral states in representations with zero third component of weak isospin). The mass terms in the Lagrangian have the following form in terms of the spinors ρ describing the current eigenstates

$$\mathcal{L}^{\nu}_{\rm mass} = -\frac{1}{2}\rho^{\rm T}\sigma_2 M\rho + {\rm h.c.}\ , \qquad M \equiv \left[\begin{array}{cc} M_{\rm L} & D \\ D^T & M_{\rm R} \end{array}\right], \tag{5.4}$$

where σ_2 is the Pauli matrix, M is a complex symmetric matrix in which the (3×3) submatrix $M_{\rm L}$ describes the masses arising in the doublet sector from (combinations of) VEVs of Higgs fields transforming as weak isotriplets, D is the $(3\times\, q)$ Dirac mass matrix coming from the VEVs of doublet Higgs fields, and $M_{\rm R}$ is a $(q\times\, q)$ matrix describing the masses in the singlet sector which are already $SU(2)_{\rm L}\otimes U(1)_Y$ invariant. The *physical* neutrino mass eigenstates, ρ_m, are then given by

$$\rho = U\rho_m\ , \tag{5.5}$$

where U is a unitary matrix which diagonalizes M in terms of the $3+q$ physical masses. Written out explicitly in the notation used by experimentalists, this says that the (LH) weak flavour eigenstates $\nu_{\alpha{\rm L}}$ $(\alpha={\rm e},\mu,\tau)$ which appear in the weak interaction coupled via W to e,μ,τ, are in general related to the mass eigenstates $\nu_{i{\rm L}}$ $(i=1,2,\ldots 3+q)$ through a leptonic Cabibbo-Kobayashi-Masakawa (CKM) mixing

$$\nu_{\alpha{\rm L}} = \Sigma_i U_{\alpha i}\nu_{i{\rm L}}\ . \tag{5.6}$$

(Henceforth the subscript $_{\rm L}$ will be implied unless otherwise specified.) This allows flavour-changing processes such as neutrino oscillations, when the neutrino mass differences are very small relative to the momenta so they propagate coherently, and neutrino decays, when the mass differences are sufficiently large that the propagation is incoherent (see Bilenky and Petcov 1987, Oberauer and von Feilitzsch 1992).

Before proceeding to study the effects of neutrino oscillations and decays, we review recent studies of the effect of a large neutrino mass on BBN (Kolb *et al* 1991, Dolgov and Rothstein 1993, Dodelson *et al* 1994, Kawasaki *et al* 1994). In (5.1) we have set $g_\nu = 2$ since only LH neutrinos (and RH antineutrinos) have full-strength weak interactions. If neutrinos have Dirac masses, then the non-interacting RH neutrino (and LH antineutrino) states can also come into thermal equilibrium through spin-flip scattering at sufficiently high temperatures, thus doubling g_ν.† The rate at which the RH states are populated is $\propto (m_\nu/T)^2$, hence Shapiro *et al* (1980) had concluded that equilibrium would *not* be achieved for a mass of $\approx 30\,\mathrm{eV}$ which was indicated at that time for the ν_e (and which is of order the cosmological bound (5.1) for stable neutrinos). However given the weaker mass limits for the other neutrinos, as also the possibility that these may be *unstable*, one must consider whether their RH states may have been populated during nucleosynthesis. Obviously this will further speed up the expansion and be in conflict with the bound on N_ν, hence an upper limit on the Dirac mass can be derived by requiring that the spin-flip scattering rate fall behind the Hubble expansion rate *before* the quark-hadron phase transition. Then the RH states do not share in the entropy release and are diluted adequately (Fuller and Malaney 1991, Enqvist and Uibo 1993). Dolgov *et al* (1995) also include the production of wrong-helicity states through $\gamma\gamma \to \pi^0 \to \nu\bar{\nu}$ (Lam and Ng 1991) as well as $\pi^\pm \to \mu\nu_\mu$, which are insensitive to T_c^qh. An updated and corrected calculation (Fields *et al* 1996) yields the constraints:

$$m_{\nu_\mu} < 310\,\mathrm{keV}\ , \qquad m_{\nu_\tau} < 370\,\mathrm{keV}\ , \tag{5.7}$$

imposing $N_\nu < 4$ and taking $T_\mathrm{c}^\mathrm{qh} > 100\,\mathrm{MeV}$. These bounds improve (degrade) by a factor of about 2, if the BBN limit on N_ν is tightened (weakened) to 3.3 (4.5).

When the neutrino mass exceeds a few MeV, they are non-relativistic at decoupling so their relic energy density (4.16) falls inversely as the self-annihilation cross-section, hence decreases with increasing mass (for $m_\nu \ll m_W$). The constraint on the speed-up rate (or on N_ν) during BBN then implies a *lower* bound on m_ν.† Initial calculations suggested a lower limit *above* the present experimental bound on m_{ν_τ}, e.g. Kolb *et al* (1991) quoted $m_\nu > 25\,\mathrm{MeV}$ corresponding to the constraint $N_\nu < 3.4$ while Dolgov and Rothstein (1993) obtained $m_\nu > 35\,\mathrm{MeV}$ for $N_\nu < 3.6$. Recently Hannestad and Madsen (1996) claimed that careful solution of the Boltzmann equation (including scattering reactions) lowers the relic abundance substantially, thus opening an allowed region for a Majorana ν_τ above 16 MeV (adopting various abundance bounds which effectively imply $N_\nu < 3.4$). As we have discussed in section 4.1, such restrictive bounds are no longer justified by the data, hence this conclusion is unreliable. Moreover for such massive neutrinos, there are additional effects on the abundances since their annihilations create electron neutrinos, viz. $\nu_\tau\bar{\nu}_\tau \to \nu_\mathrm{e}\bar{\nu}_\mathrm{e}$, which can bias neutron-proton interconversions.

† Actually in most extensions of the Standard Model wherein neutrinos have masses, these are associated with the violation of global lepton number and are Majorana in nature, so this process is irrelevant. Even for Dirac neutrinos, the RH states are not populated at decoupling for a mass of $\mathrm{O}(100)\,\mathrm{eV}$, hence it is always valid to take $g_\nu = 2$ in (5.1).

† This implicitly assumes that the nucleon-to-photon ratio is unaffected by the massive neutrino, so that the N_ν bound is directly applicable. However the neutrino *must* decay subsequently and the entropy thus released will in fact lower η (see section 4.2.1). Therefore these analyses are only valid if the decays create *no* entropy at all, which is rather unnatural.

Taking such effects into account excludes the high mass window, even for a relaxed upper bound on N_ν (Dolgov *et al* 1996).

For any of the above bounds to be valid, the neutrino must be present at the time of nucleosynthesis, i.e. its lifetime must exceed $\sim 1-1000\,\mathrm{sec}$. However the present (redshifted) energy density of the decay products may be excessive unless the decays occur early enough (Dicus *et al* 1977). Making the conservative assumptions that the decay products are all massless and that their energy-density has always dominated the universe, we obtain the bound†

$$\tau_\nu \lesssim \left(\frac{m_\nu}{94\,\mathrm{eV}}\frac{g_\nu}{2}\right)^{-2}(\Omega h^2)^2 t_0 \lesssim 3\times 10^{12}\,\mathrm{sec}\left(\frac{m_\nu}{10\,\mathrm{keV}}\frac{g_\nu}{2}\right)^{-2}, \tag{5.8}$$

using the inequality $\Omega\, h^2 < 1/3$ for $t_0 > 10^{10}\,\mathrm{yr}$, $h > 0.4$ which holds for a radiation-dominated universe (Pal 1983). We will see below that this lifetime bound *cannot* be satisfied by neutrinos allowed by the mass bound (5.7), unless they have 'invisible' decays into hypothetical Goldstone bosons. However a strict upper bound on the mass of the ν_τ (5.13) can still be obtained (irrespective of whether it is Dirac or Majorana) by considering its decays into Standard Model particles, as we discuss below.

5.1.2. Neutrino decays: The most studied decay mode for neutrinos has been the radiative process $\nu_i \to \nu_j\gamma$ and the consequences of such decays, both in astrophysical sites of neutrino production and in the early universe, have been widely investigated (see Maalampi and Roos 1990, Sciama 1993). In the Standard Model extended to allow for Dirac neutrino masses (without unpaired singlets) this decay mode is severely supppressed by a leptonic Glashow-Iliopoulos-Maiani (GIM) mechanism and consequently has a rather long lifetime (Marciano and Sanda 1977, Petcov 1977, see Pal and Wolfenstein 1982)

$$\tau_{\nu_i\to\nu_j\gamma} \simeq \frac{2048\pi^4}{9\alpha G_\mathrm{F}^2 m_{\nu_i}^5}\frac{1}{|\sum_\alpha U^*_{\alpha j}U_{\alpha i}F(r_\alpha)|^2} > 2.4\times 10^{14}\,\mathrm{sec}\left(\frac{m_{\nu_i}}{\mathrm{MeV}}\right)^{-5}, \tag{5.9}$$

where $r_\alpha \equiv (m_{\ell_\alpha}/M_W)^2$, $F(r_\alpha) \simeq -\frac{3}{2}+\frac{3}{4}r_\alpha$ for $r_\alpha \ll 1$ (i.e. for $\ell_\alpha = \tau$, and we have assumed $m_{\nu_i} \gg m_{\nu_j}$). Nieves (1983) has noted that the next-order process $\nu_i \to \nu_j\gamma\gamma$ is not GIM suppressed and may therefore possibly dominate over single photon decay. For $m_{\nu_j} \ll m_{\nu_i} \ll m_\mathrm{e}$,

$$\tau_{\nu_i\to\nu_j\gamma\gamma} = \frac{552960\pi^5 m_\mathrm{e}^4}{\alpha^2 G_\mathrm{F}^2 m_{\nu_i}^9}\frac{1}{|U^*_{\mathrm{e}j}U_{\mathrm{e}i}|^2} > 1.1\times 10^{12}\,\mathrm{sec}\,|U_{\mathrm{e}i}|^{-2}\left(\frac{m_{\nu_i}}{\mathrm{MeV}}\right)^{-9}. \tag{5.10}$$

† This assumes that all neutrinos decay instantaneously at $t = \tau_\nu$; numerical integration over an exponential distribution of decay times relaxes the bound on the lifetime by about 50% (Dicus *et al* 1978a, Massó and Pomarol 1989). Note that the bound shown by Kolb and Turner (1990) is incorrect.

However, when $m_{\nu_i} > m_{\rm e}$, the lifetime increases to (Sarkar and Cooper 1984):

$$\tau_{\nu_i \to \nu_j \gamma\gamma} = \frac{384\pi^5}{\alpha^2 G_{\rm F}^2 m_{\nu_i}^5} \frac{1}{|U_{{\rm e}j}^* U_{{\rm e}i}|^2} > 1.1 \times 10^{10}\,{\rm sec}\,|U_{{\rm e}i}|^{-2} \left(\frac{m_{\nu_i}}{\rm MeV}\right)^{-5} . \qquad (5.11)$$

In any case, for $m_{\nu_i} \gtrsim 2m_{\rm e}$, the tree-level charged current decay $\nu_i \to {\rm e^-e^+}\nu_{\rm e}$ takes over, with the lifetime

$$\tau_{\nu_i \to {\rm e^-e^+}\nu_{\rm e}} = \frac{192\pi^3}{G_{\rm F}^2 m_{\nu_i}^5} \frac{1}{|U_{{\rm e}i}|^2 f(m_{\rm e}/m_{\nu_i})} = 2.4 \times 10^4\,{\rm sec}\,|U_{{\rm e}i}|^{-2} \left(\frac{m_{\nu_i}}{\rm MeV}\right)^{-5} , \qquad (5.12)$$

for $m_{\nu_i} \gg m_{\rm e}$ where $f(x)$ is a phase-space factor ($\simeq 1$ for $x \ll 1$).

Experiments at PSI and TRIUMF have set upper limits on the mixing $|U_{{\rm e}i}|^2$ of any ν_i with mass in the range $4-54$ MeV which can be emitted along with an electron in pion decay (see Bryman 1993), by measuring the branching ratio $R_\pi = (\pi \to {\rm e}\nu)/(\pi \to \mu\nu)$ and/or searching for additional peaks in the energy spectrum of $\pi \to {\rm e}\nu$ decays (e.g. Bryman *et al* 1983, Azeluos *et al* 1986, De Leener-Rosier *et al* 1991, Britton *et al* 1992, 1994). Similar, although less stringent bounds are obtained from studies of kaon decay (Yamazaki *et al* 1984). Direct searches have been also carried out for decays of heavier neutrinos with masses upto a few GeV produced through mixing in accelerator beams of muon and electron neutrinos as well as for unstable tau neutrinos produced through decays of D_s charmed mesons in 'beam-dump' experiments (Bergsma *et al* 1983, Cooper-Sarkar *et al* 1985, Bernardi *et al* 1986). Searches have also been carried out for radiative decays of electron and muon neutrinos (e.g. Oberauer *et al* 1987, Krakauer *et al* 1991) in low energy reactor and accelerator beams.

As we have discussed in section 4.2, primordial nucleosynthesis restricts such decays in several distinct ways. The most general is the constraint (4.22) on entropy generation subsequent to nucleosynthesis which imposes an upper bound on the lifetime given the relic energy density of the decaying neutrino as a function of its mass (Sato and Kobayashi 1977, Miyama and Sato 1978). Using (5.12) this can be converted into a *lower* limit on the mixing $|U_{{\rm e}i}|^2$ of a massive neutrino. Kolb and Goldman (1979) noted that the limit thus extracted from the lifetime bound obtained from consideration of the D abundance by Dicus *et al* (1978b) is *higher* than the upper limit on this mixing as deduced from π and K decays, if $m_\nu \lesssim 9$ MeV. This conclusion was shown (Sarkar and Cooper 1984) to hold even using the less restrictive lifetime bound (4.22) following from the more reliable constraint $Y_{\rm p}(^4{\rm He}) < 0.25$, but using improved experimental limits on the mixing (Bryman *et al* 1983). For Majorana neutrinos which have a higher relic abundance, Krauss (1983b) found (using the Dicus *et al* (1978b) lifetime bound) that any neutrino mass below 23 MeV was ruled out on the basis of this argument. A similar conclusion was arrived at by Terasawa *et al* (1988) who adopted the even more generous bound $Y_{\rm p}(^4{\rm He}) < 0.26$. The present situation is illustrated in figure 16 which shows the upper bound on τ_ν inferred from figure 13(a) (corresponding to the requirement $Y_{\rm p}(^4{\rm He}) < 0.25$), where we have calculated the relic neutrino abundance assuming it is a Dirac particle. For m_ν less than about 15 MeV these are *below* the lower bound to the lifetime calculated from the best current limits on the mixing $|U_{{\rm e}i}|^2$ (Britton *et al* 1992, 1994, De Leener-Rosier *et al* 1991). Recently Dodelson *et al* (1994) have made a comprehensive study of the lifetime bounds on an unstable neutrino, taking into account many (small) effects ignored in previous calculations. They adopt the more restrictive bound $Y_{\rm p}(^4{\rm He}) < 0.24$, which leads to more stringent constraints than those obtained previously, extending down to a lifetime of $\mathrm{O}(10^2)$ sec. However, as discussed in section 3.2.1, this bound can no longer be considered reliable.

It was believed (Lindley 1979, Cowsik 1981) that the radiative decays of neutrinos heavier than about 5 MeV can be restricted further by constraining the photofission of deuterium by the decay photons. However Kolb and Scherrer (1982) argued that this constraint does not apply to the *dominant* decay mode $\nu_i \rightarrow {\rm e}^-{\rm e}^+\nu_{\rm e}$ since the rapid thermalization of the decay $\rm e^\pm$ by scattering against the background photons severely suppresses D photofission. This is erroneous since the background photons are themselves energetic enough during the BBN era to be Compton scattered by the decay $\rm e^\pm$ to energies above the threshold for D photofission. Sarkar and Cooper (1984) calculated the spectrum of the scattered photons and concluded that the D abundance would be depleted by a factor exceeding 100 unless the neutrino decay lifetime is less than about $20 - 100$ sec for $m_\nu \sim 5 - 100$ MeV. A similar constraint was obtained by Krauss (1984). Subsequently Lindley (1985) pointed out that the scattered photons were much more likely to undergo $\gamma - \gamma$ scattering on the energetic photons in the Wien tail of the thermal background than photodissociate deuterium (see also Scherrer 1984). Taking this into account relaxes the upper bound on the lifetime by a factor of about 100 (Lindley 1985) as shown in figure 16.† Thus Krauss (1985) and Sarkar (1986) concluded that cosmological and laboratory limits appeared to allow an unstable ν_τ with a lifetime of $\mathrm{O}(10^3)$ sec and a mass between 20 MeV and its (then) upper limit of 70 MeV. Subsequently the experimental limit on the ν_τ mass has come down to 24 MeV while the laboratory limits on the mixing angle $|U_{{\rm e}i}|^2$ have improved further. As shown in figure 16, the experimental lower bound on $\tau_{\nu_i \rightarrow {\rm e}^-{\rm e}^+\nu_{\rm e}}$ now exceeds the cosmological upper bounds from D photofission and entropy generation, for a neutrino mass in the range $1 - 25$ MeV. Thus the conclusion of Sarkar and Cooper (1984), viz. that

$$m_{\nu_\tau} < 2m_{\rm e} \ , \tag{5.13}$$

is reinstated. We emphasize that this bound is more general than the similar one (5.7) which only applies to Dirac neutrinos which decay 'invisibly' after nucleosynthesis.

† Kawasaki *et al* (1986) and Terasawa *et al* (1988) also studied this constraint using numerical methods but found it to be weaker by a factor of about 2 than the semi-analytic result of Lindley.

Sarkar and Cooper (1984) had argued that a ν_τ lighter than 1 MeV has no decay modes which are fast enough to satisfy the constraint (5.8) from the energy density. The radiative decay $\nu_\tau \to \nu_e \gamma$ is generally too slow and in any case is observationally required to have a lifetime greater than the age of the universe (see Sciama 1993). The 'invisible' decay $\nu_\tau \to \nu_e \bar{\nu}_e \nu_e$ is also GIM suppressed but may be mediated sufficiently rapidly by Higgs scalars in the left-right symmetric model $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ (e.g. Roncadelli and Senjanović 1981) or through GIM-violation by flavour-changing-neutral-currents (FCNC) in other extensions of the Standard Model (e.g. De Rújula and Glashow 1980, Hosotani 1981). However, this neccessarily enhances the radiative decay mode as well (McKellar and Pakvasa 1983, Gronau and Yahalom 1984) and is thus ruled out observationally. It is thus neccessary to invent a new massless (or very light) particle for the neutrino to decay into. Since giving neutrinos masses usually involves the spontaneous violation of lepton number, a candidate particle is the associated Goldstone boson, the Majoron (Chikashige *et al* 1980, Gelmini and Roncadelli 1981). The neutrino decay lifetime in such models is usually too long (e.g. Schechter and Valle 1982) but can be made sufficiently short if the model is made contrived enough. Although Majorons which have couplings to the Z^0 are now ruled out by *LEP* , there still remain some viable models with, e.g. singlet Majorons (see Gelmini and Roulet 1995). If neutrinos can indeed have fast decays into Majorons then the BBN bounds on visible decays discussed above are not relevant. Nevertheless the constraints based on the expansion rate are still valid and additional constraints obtain if the final state includes electron (anti)neutrinos (e.g. Terasawa and Sato 1987, Kawasaki *et al* 1994, Dodelson *et al* 1994). Interestingly enough, such decays can slightly reduce the ^{4}He abundance for a mass of $O(1)$ MeV and a lifetime of $O(1)$ sec, thus *weakening* the bound on N_ν.

5.1.3. Neutrino oscillations: Neutrino *flavour* oscillations are not relevant in the early universe since the number densities of all flavours are equal in thermal equilibrium and all three species decouple at about the same temperature. However if there is mixing between the left-handed (active) and right-handed (sterile) neutrinos, then oscillations may bring these into thermal equilibrium boosting the expansion rate while depleting the population of active (electron) neutrinos which participate in nuclear reactions. Thus powerful bounds on such mixing can be deduced from consideration of the effects on nucleosynthesis. These considerations are particularly relevant to reports of experimental anomalies attributed to the existence of sterile neutrinos, e.g. the 17 keV anomaly in β-decay (Hime and Jelly 1991, see Hime *et al* 1991) and the recently reported 33.9 MeV anomaly in π decay (Armbruster *et al* 1995, see Barger *et al* 1995)

The first estimates of these effects (e.g. Khlopov and Petcov 1981, Fargion and Shepkin 1981, Langacker *et al* 1986, Manohar 1987) did not take into account the coherent forward scattering of the active species (Nötzold and Raffelt 1988) which provides a correction to the average momentum ($\langle p_{\rm e}\rangle = 3.15T$) of $\nu_{\rm e}$ in the thermal plasma. For $1\,{\rm MeV} \lesssim T \ll 100\,{\rm MeV}$, this is

$$V_{\rm e} = \sqrt{2}G_{\rm F}n_\gamma\left(L - A\frac{T^2}{M_W^2}\right) \ , \tag{5.14}$$

where $A = 4(1+0.5\cos^2\theta_{\rm W})(7\zeta(4)/2\zeta(3))^2 = 55$ and L is a sum of terms proportional to the lepton and baryon asymmetries in the plasma (and therefore appears with opposite sign in the corresponding effective energy for $\bar{\nu_{\rm e}}$). In the presence of neutrino oscillations, the lepton asymmetry, if not too large, is dynamically driven to zero on a time-scale large compared to the oscillation time (Enqvist *et al* 1990a, 1991); thus the average self-energy correction is $\langle V_{\rm e}\rangle = V_{\rm e}(L = 0)$.† Hence a Mikheyev-Smirnov-Wolfenstein (MSW) resonant transition of $\nu_{\rm e}$ into $\nu_{\rm s}$ (and *simultaneously* $\bar{\nu_{\rm e}}$ into $\bar{\nu_{\rm s}}$) can occur satisfying $V_{\rm e} = \Delta m^2\cos 2\theta_{\rm v}/\langle p_{\rm e}\rangle$, only if the mass-difference squared, $\Delta m^2 \equiv m^2_{\nu_{\rm s}} - m^2_{\nu_{\rm e}}$, is *negative*, i.e. opposite to the case in the Sun. If this occurs after electron neutrino decoupling (at about 2 MeV) but before neutron freeze-out is complete (at about 0.2 MeV), then the surviving neutron fraction is larger, leading to increased helium production.‡ Enqvist *et al* (1990b) have examined these effects using the semi-analytic formulation of Bernstein *et al* (1989) and conclude that the survival probability $P(\theta_{\rm v})$ of $\nu_{\rm e}/\bar{\nu_{\rm e}}$ must exceed 0.84 in order not to alter $Y_{\rm p}(^4{\rm He})$ by more than 4%. Using the Landau-Zener formula for the probability of transition between adiabatic states, they derived a severe bound on the vacuum mixing angle $\theta_{\rm v}$.

The case when Δm^2 is positive is more interesting, having been proposed in the context of solutions to the Solar neutrino problem (see Bahcall 1989). Here the major effect is that the sterile neutrinos, which nominally decouple at a very high temperature and thus have a small abundance relative to active neutrinos, $n_{\nu_{\rm s}}/n_{\nu_{\rm a}} \approx 0.1$ (Olive and Turner 1982), can be brought back into thermal equilibrium through $\nu_{\rm a} - \nu_{\rm s}$ oscillations. The production rate (through incoherent scattering) is

$$\Gamma_{\nu_{\rm s}} \simeq \frac{1}{2}(\sin^2 2\theta_{\rm m})\ \Gamma_{\nu_{\rm a}} \ , \tag{5.15}$$

where $\Gamma_{\nu_{\rm a}}$ is the total interaction rate of the active species ($\Gamma_{\nu_{\rm e}} \simeq 4G_{\rm F}^2T^5, \Gamma_{\nu_{\mu,\tau}} = 2.9G_{\rm F}^2T^5$) and the mixing angle in matter is related to its vacuum value as

$$\sin^2 2\theta_{\rm m} = (1 - 2x\cos 2\theta_{\rm v} + x^2)^{-1}\sin^2 2\theta_{\rm v} \ , \tag{5.16}$$

† However for large neutrino degeneracy the first term in (5.14) dominates and oscillations between different active flavours becomes the important process (Savage *et al* 1991). If the degeneracy is as large as $L \gtrsim 10^{-5}$, the constraints on active-sterile mixing are evaded (Foot and Volkas 1995a); such an asymmetry can arise due to the oscillations themselves (Foot *et al* 1996).

‡ In fact, resonant transitions of $\nu_{\rm e}$ to the (very slightly cooler) ν_μ or ν_τ in this temperature interval can have a similar but smaller effect; $Y_{\rm p}$ is increased by at most 0.0013 (Langacker *et al* 1987).

where $x \equiv 2\langle p\rangle\langle V\rangle/\Delta m^2$. (One requires $\sin^2 2\theta_{\rm m} < 0.15$ to be able to ignore non-linear feedback processes which would reduce $\Gamma_{\nu_{\rm s}}$.) The ratio of the sterile neutrino production rate to the Hubble rate (2.64), $\Gamma_{\nu_{\rm s}}/H$, thus has a maximum at

$$T_{\rm max} = B_{\nu_{\rm a}}(\Delta m^2)^{1/6} , \tag{5.17}$$

where $B_{\nu_{\rm e}} = 10.8$ and $B_{\nu_{\mu,\tau}} = 13.3$. If $\Gamma_{\nu_{\rm s}}/H > 1$ at this point, then the sterile neutrinos will be brought into equilibrium thus boosting the expansion rate, hence the synthesized helium abundance. The BBN bound on N_ν can now be translated into an upper bound on the mixing. For example, Barbieri and Dolgov (1990, 1991) used $N_\nu < 3.8$ (which requires $T_{\rm max} > m_\mu$ (see (4.4)) to obtain $(\sin\theta_{\rm v})^4\Delta m^2 < 6\times 10^{-3}{\rm eV}^2$ while Kainulainen (1990) used $N_\nu < 3.4$ (which requires $T_{\rm max} > T_{\rm c}^{\rm qh}$) to obtain $(\sin\theta_{\rm v})^4\Delta m^2 < 3.6\times 10^{-4}{\rm eV}^2$. (Somewhat weaker bounds obtain for ν_μ/ν_τ oscillations into singlets.) Barbieri and Dolgov (1990) also noted that if $\nu_{\rm e}-\nu_{\rm s}$ oscillations occur *after* electron neutrinos decouple, then their number density is depleted, giving rise to a (negative) neutrino chemical potential which increases the helium abundance (see (3.40)). Then the bound $N_\nu < 3.8$ corresponds to the excluded region $\sin^2 2\theta_{\rm v} \gtrsim 0.4$ and $\Delta m^2 \gtrsim 2\times 10^{-7}{\rm eV}^2$.

Recently Enqvist *et al* (1992b) have performed a thorough examination of both cases, improving on approximations made in the earlier estimates of the collision rates through detailed calculations. (In fact we have quoted above their values for $\Gamma_{\nu_{\rm a}}$ and $B_{\nu_{\rm a}}$.) They consider several possible constraints from nucleosynthesis, viz. $N_\nu < 3.1, 3.4, 3.8$ and perform numerical calculations to determine the allowed parameters in the $\Delta m^2 - \sin^2 2\theta_{\rm v}$ plane. In contrast to the previous results, these authors find that BBN considerations *rule out* the large mixing-angle MSW solution to the Solar neutrino problem. They also consider $\nu_\mu - \nu_{\rm s}$ mixing and show that this cannot be a solution to the atmospheric neutrino anomaly (see Beier *et al* 1992, Perkins 1993). Their results are confirmed by the similar calculations of Shi *et al* (1993). Of course all these results are *invalidated* if the bound on N_ν is relaxed to exceed 4, which we have argued (section 4.1) is allowed by the observational data (Cardall and Fuller 1996b, Kernan and Sarkar 1996b).

These bounds have been discussed (e.g. Dixon and Nir 1991, Babu and Rothstein 1991, Enqvist *et al* 1992c,d, Cline 1992, Cline and Walker 1992) in connection with the 17 keV neutrino which was seen to be emitted in β-decay with a mixing of about 1% with the electron neutrino (Simpson 1985, see Hime 1992). The most likely interpretation of this state was that it was either a singlet neutrino or else a member of a pseudo-Dirac pair. The theoretical possibilities as well as the constraints from nucleosynthesis (and other cosmological/astrophysical arguments) have been comprehensively reviewed by Gelmini *et al* (1992). However the experimental evidence now disfavours the existence of this particle (see Particle Data Group 1994), the signal for which was faked by a conspiracy of systematic errors (Bowler and Jelley 1994). With regard to the KARMEN anomaly (Armbruster *et al* 1994) which has been interpreted as due a singlet neutrino of mass 33.9 MeV mixing with all three doublet neutrinos (Barger *et al* 1995), the mixing angles are not known but are restricted within certain limits e.g. $|U_{ex}|^2 < 8.5 \times 10^{-7}$, $|U_{\mu x}|^2 < 2 \times 10^{-3}$, $|U_{\tau x}|^2 < 1$. If the mixing is sufficiently large, the x particle would be brought into equilibrium at a temperature of a few GeV. Although its abundance would thus be suppressed relative to doublet neutrinos by the entropy production in the quark-hadron transition, it would still have a large energy density during BBN since it would have become *non-relativistic* by then. The x decays can cause photofission of the synthesized abundances (Langacker *et al* 1986) so the mixing angles are required to be large enough that such decays occur sufficiently early, obeying the cosmological mass-lifetime constraints shown in figure 16. Such constraints have also been discussed in connection with hypothetical sterile neutrinos having masses larger than a GeV (Bamert *et al* 1995).

5.1.4. Neutrino magnetic moments: A massless neutrino has no electromagnetic properties but when the Standard Model is extended to include a Dirac neutrino mass, this generates a magnetic dipole moment (e.g. Lee and Schrock 1977)

$$\mu_\nu = \frac{3eG_{\rm F}}{8\sqrt{2}\pi^2} = 3.2 \times 10^{-19} \left(\frac{m_\nu}{\rm eV}\right) \mu_{\rm B} \ , \tag{5.18}$$

A Majorana neutrino, being its own antiparticle, has zero magnetic (and electric) dipole moments by CPT invariance. This refers to the *diagonal* moments; in general flavour-changing transition magnetic (and electric) moments exist for both Dirac and Majorana neutrinos. The neutrino magnetic moment may be significantly enhanced over the above estimate in extensions of the SM (see Pal 1992).

A Dirac magnetic moment allows the inert RH states to be produced in the early universe through $\nu_{\rm L}{\rm e} \to \nu_{\rm R}{\rm e}$ scattering (Morgan 1981a) with cross-section

$$\sigma_{\nu_{\rm L}{\rm e}\to\nu_{\rm R}{\rm e}} = \pi \left(\frac{\alpha}{m_{\rm e}}\right)^2 \left(\frac{\mu_\nu}{\mu_{\rm B}}\right)^2 \ln\left(\frac{q^2_{\rm max}}{q^2_{\rm min}}\right) \ , \tag{5.19}$$

Fukugita and Yazaki (1987) noted that $q_{\rm max} = 3.15T$ whereas $q_{\rm min} \sim 2\pi/l_{\rm D}$ where $l_{\rm D} = (T/4\pi\, n_{\rm e}\alpha)^{1/2}$ is the Debye length in the plasma of electron density $n_{\rm e}$. By the arguments of section 4.1, the $\nu_{\rm R}$ should go out of equilibrium early enough that its abundance is adequately diluted by subsequent entropy generation. This requires $\mu_\nu \lesssim 1.5 \times 10^{-11}\mu_{\rm B}$ according to the approximate calculation of Morgan (1981a). A more careful analysis (Fukugita and Yazaki 1987) gives

$$\mu_\nu < 5 \times 10^{-11}\mu_{\rm B} \left(\frac{T_{\rm c}^{\rm qh}}{200\,{\rm MeV}}\right)^2 , \tag{5.20}$$

corresponding to the usual constraint $N_\nu < 4$. (The conservative constraint $N_\nu < 4.5$ would not change this significantly.) This is more stringent than direct experimental bounds, e.g. $\mu_{\nu_e} < 1.8 \times 10^{-10} \mu_B$ (Derbin *et al* 1994) and $\mu_{\nu_\mu} < 1.7 \times 10^{-9} \mu_B$ (Krakauer *et al* 1990). If there is a primordial magnetic field B then spin-procession can further populate the RH states (e.g. Shapiro and Wasserman 1981), leading to the correlated bound $\mu_\nu < 10^{-16} \mu_B (B/10^{-9}\mathrm{G})^{-1}$ (Fukugita *et al* 1988).

Giudice (1990) pointed out that the bound (5.20) applies only to neutrinos which are relativistic at nucleosynthesis and can therefore be evaded by the tau neutrino which is experimentally allowed to have a mass upto 24 MeV. Indeed whereas such a massive ν_τ would nominally have too high a relic abundance, a magnetic moment of $\mathrm{O}(10^{-6})\mu_B$ would enable it to self-annihilate rather efficiently (through γ rather than Z^0 exchange) so as to make $\Omega_{\nu_\tau} \sim 1$ today. There is no conflict with nucleosynthesis since the ν_τ energy density during BBN can be much less than that of a relativistic neutrino. In fact Kawano *et al* (1992) calculate that if the magnetic moment of a MeV mass ν_τ exceeds

$$\mu_{\nu_\tau} \gtrsim 7 \times 10^{-9} \mu_B , \tag{5.21}$$

its energy density during BBN is sufficiently reduced that it satisfies $Y_p(^4\mathrm{He}) < 0.24$ (see also Grasso and Kolb 1996). (For a much larger μ_{ν_τ}, the self-annihilation of ν_τs is so efficient that effectively $N_\nu \simeq 2$ rather than 3, hence the ^{4}He abundance is actually reduced, rather than increased, relative to standard BBN!) However the direct experimental bound $\mu_{\nu_\tau} < 5.4 \times 10^{-7} \mu_B$ rules out the possibility that a MeV mass tau neutrino can constitute the dark matter (Cooper-Sarkar *et al* 1992).

5.1.5. New neutrino interactions: Apart from a magnetic moment, neutrinos may have additional interactions in extensions of the Standard Model and this can be constrained by BBN in a similar manner (Hecht 1971, Morgan 1981b). For example, Grifols and Massó (1987) have calculated a bound on the neutrino charge-radius† defined through the expression

$$\langle r^2 \rangle \equiv \left(\frac{\sigma_{e^+e^- \to \nu_R \nu_R}}{\pi \alpha^2 q^2 / 54} \right)^{1/2} < 7 \times 10^{-33}\ \mathrm{cm}^2 , \tag{5.22}$$

corresponding to the constraint $N_\nu < 4$. Massó and Toldrà (1994a) have also considered a hypothetical vector-type interaction *between* neutrinos, $\mathcal{H} = F_V (\bar{\nu}_i \gamma^\mu \nu_i)(\bar{\nu}_j \gamma^\mu \nu_j)$, which can bring RH states into equilibrium. Requiring as before that this does not happen below the quark-hadron transition implies the limit

$$F_V < 3 \times 10^{-3} G_F . \tag{5.23}$$

† However, this is better interpreted as a bound on the scattering cross-section (mediated through any process) since the neutrino charge-radius is *not* a gauge-invariant quantity (Lee and Shrock 1977).

Another BBN constraint on non-standard interactions was derived by Babu *et al* (1991).

Kolb *et al* (1986c) have studied hypothetical 'generic' interacting species, viz. particles which maintain good thermal contact with neutrinos (or photons) throughout the BBN epoch. They show that the effect on BBN depends on the particle mass and cannot be simply parametrized in terms of ΔN_ν. An example is a massive neutrino in the triplet-Majoron model (Gelmini and Roncadelli 1981) which maintains equilibrium with light neutrinos through exchange of Majorons — the Goldstone boson associated with global lepton number violation. Stringent bounds are then imposed on the Majoron couplings; however this model has in any case been experimentally ruled out by *LEP* .

Interactions mediated by new gauge bosons will be considered in the context of extended technicolour (section 5.2) and superstring-motivated models (§ 5.3.4).

5.2. Technicolour

This is an attractive mechanism for spontaneously breaking the electroweak $SU(2)_\mathrm{L} \otimes U(1)_Y$ symmetry *non-perturbatively*, without introducing fundamental Higgs bosons. It does so in a manner akin to the breaking of the $SU(2)_\mathrm{L} \otimes SU(2)_\mathrm{R}$ chiral symmetry of the (nearly) massless u and d quarks by the formation of a $q\bar{q}$ condensate at Λ_QCD when the $SU(3)_\mathrm{c}$ colour force becomes strong (see Farhi and Susskind 1981). A generic technicolour scenario thus invokes new hyper-strong interactions with an intrinsic scale of $\Lambda_\mathrm{TC} \approx 0.5\,\mathrm{TeV}$, due to gauge interactions with $N_\mathrm{TC} \geq 3$ unbroken technicolours. These interactions bind techniquarks Q_T in the fundamental N_TC representation of $SU(N_\mathrm{TC})$, forming $Q_\mathrm{T}\bar{Q}_\mathrm{T}$ 'technimeson' and $Q_\mathrm{T}^{N_\mathrm{TC}}$ 'technibaryon' bound states. The latter will have integer spin if N_TC is even, and the choice often favoured is $N_\mathrm{TC} = 4$. In this case the lightest technimeson would be expected to be short-lived with $\tau \ll 1$ sec, thus evading BBN constraints, but the lightest technibaryon, which has a mass

$$m_\mathrm{TB} \simeq m_\mathrm{p} \left(\frac{\Lambda_\mathrm{TC}}{\Lambda_\mathrm{QCD}}\right) = m_\mathrm{p} \left(\frac{v}{f_\pi}\right) = 2.4\,\mathrm{TeV}\ , \tag{5.24}$$

is likely to be metastable, by analogy with the proton of QCD. Indeed as in QCD, there is no renormalizable interaction that can cause technibaryon decay. However, the minimal technicolour model must in any case be extended to incorporate quark and lepton masses, and one might anticipate that it is unified in some kind of techni-GUT. Therefore one expects, in general, higher-order effective non-renormalizable interactions which cause technibaryon decay, of the form

$$\mathcal{L}_\mathrm{ETC} = \frac{Q_\mathrm{T}^{N_\mathrm{TC}} f^n}{\Lambda_\mathrm{ETC}^{3/2(N_\mathrm{TC}+n)-4}}\ , \tag{5.25}$$

where f is a quark or lepton field and $\Lambda_{\rm ETC}$ is some mass scale $\gg \Lambda_{\rm TC}$ at which the effective interaction is generated. These would imply a technibaryon lifetime

$$\tau_{\rm TB} \simeq \frac{1}{\Lambda_{\rm TC}} \left(\frac{\Lambda_{\rm ETC}}{\Lambda_{\rm TC}}\right)^{3(N_{\rm TC}+n)-8} , \tag{5.26}$$

i.e. $\sim 10^{-27}(\Lambda_{\rm ETC}/\Lambda_{\rm TC})^4$ sec, for the favoured case $N_{\rm TC} = 4$ with the minimal choice $n = 0$.

Estimating the self-annihilation cross-section of technibaryons to be (e.g. Chivukula and Walker 1990)

$$\langle\sigma v\rangle_{\rm TB} \simeq \langle\sigma v\rangle_{\rm p\bar{p}} \left(\frac{m_{\rm p}}{m_{\rm TB}}\right)^2 \simeq 3\times10^{-5}{\rm GeV}^2, \tag{5.27}$$

the *minimum* expected relic abundance is $m_{\rm TB}\, n_{\rm TB}/n_\gamma = 3\times10^{-13}\,{\rm GeV}$, where we have taken account of entropy generation following freeze-out at $\approx 70\,{\rm GeV}$. Unstable technibaryons with such a small abundance are not constrained by nucleosynthesis (Dodelson 1989). However technibaryons may have a much higher relic density if they possess an asymmetry of the same order as the baryon asymmetry (Nussinov 1985). If the latter is due to a net $B-L$ generated at some high energy scale, then this would be subsequently distributed among *all* electroweak doublets by fermion-number violating processes in the Standard Model at temperatures above the electroweak scale (see Shaposhnikov 1991, 1992), thus naturally generating a technibaryon asymmetry as well. If such $B+L$ violating processes cease being important below a temperature $T_* \simeq T_c^{\rm EW} \approx 300\,{\rm GeV}$, then the technibaryon-to-baryon ratio, which is suppressed by a factor $[m_{\rm TB}(T_*)/T_*]^{3/2}{\rm e}^{-m_{\rm TB}(T_*)/T_*}$, is just right to give $\Omega_{\rm TB} \simeq 1$ (Barr *et al* 1990), i.e. $m_{\rm TB}\, n_{\rm TB}/n_\gamma = 3\times10^{-8}\,{\rm GeV}$. As can be seen from figures 13 and 15, lifetimes between $\approx 1\,{\rm sec}$ and $\approx 10^{13}\,{\rm sec}$ are forbidden for particles with such an abundance. If technibaryons decay with a longer lifetime, i.e. after (re)combination, their decay products would be directly observable today. Constraints from the diffuse gamma ray background (Dodelson 1989) as well as the diffuse high energy neutrino background (Gondolo *et al* 1993) then extend the lower lifetime bound all the way to $\approx 3\times10^{17}\,{\rm yr}$ (Ellis *et al* 1992), i.e. such particles should be essentially stable and an important component of the dark matter.† These lifetimes bounds imply that

$$\Lambda_{\rm ETC} \lesssim 6\times10^9\ {\rm GeV} \quad {\rm or} \quad \Lambda_{\rm ETC} \gtrsim 10^{16}\ {\rm GeV}\ . \tag{5.28}$$

The former case would be applicable to any extended technicolour (ETC) model containing interactions that violate technibaryon number, while the latter case could accomodate a techni-GUT at the usual grand unification scale.

† Although technibaryons with masses upto a few TeV are experimentally ruled out as constituents of the Galactic dark matter if they have *coherent* weak interactions (e.g. Ahlen *et al* 1987, Caldwell *et al* 1988, Boehm *et al* 1991), the *lightest* technibaryon may well be an electroweak singlet (as well as charge and colour neutral), thus unconstrained by such direct searches (e.g. Chivukula *et al* 1993).

Another constraint on ETC models follows from the BBN bound on N_ν. Krauss *et al* (1993) note that such models typically contain right-handed neutrinos (see King 1995) which can be produced through $\nu_{\rm L}\bar{\nu_{\rm L}} \to \nu_{\rm R}\bar{\nu_{\rm R}}$ which proceeeds through the exchange of ETC gauge bosons of mass $M_{\rm ETC}$. Thus one should require this process to go out of equilibrium before the quark-hadron phase transition (§ 4.1). Krauss *et al* (1995) adopt the bound $N_\nu \lesssim 3.5$ to derive

$$\frac{M_{\rm ETC}}{g_{\rm ETC}} \gtrsim 2 \times 10^4\,{\rm GeV}\ , \tag{5.29}$$

where $g_{\rm ETC}$ is the relevant ETC gauge coupling. While the above cosmological constraints on technicolour are not particularly restrictive, the basic idea has in any case fallen into disfavour because of the difficulties in constructing realistic phenomenological models which are consistent with experimental limits on flavour-changing neutral currents and light technipion states (see King 1995). Also the radiative corrections to SM parameters are generally expected to be large (see Lane 1993), in conflict with the experimental data (see Langacker 1994).

5.3. Supersymmetry and supergravity

Because of such experimental difficulties with dynamical electroweak symmetry breaking, it is now generally accepted that the problems associated with a fundamental Higgs boson are better cured by supersymmetry, in a manner consistent with all such experimental constraints (see Adriani *et al* 1993, Baer *et al* 1995). As noted earlier, the quadratically divergent radiative corrections to the mass of a fundamental Higgs scalar can be cancelled by postulating that for every known fermion (boson), there is a boson (fermion) with the *same* interactions. Thus each particle of the Standard Model must be accompanied by its superpartner — spin-$\frac{1}{2}$ partners for the gauge and Higgs bosons and spin-0 partners for the leptons and quarks — in the minimal supersymmetric Standard Model (see Nilles 1984, Haber and Kane 1985). The Lagrangian consists of a supersymmetric part with gauge interactions as in the SM while the Yukawa interactions are derived from the 'superpotential'

$$P_{\rm MSSM} = h_u Q H_2 u^c + h_d Q H_1 d^c + h_e L H_1 e^c + \mu H_1 H_2\ . \tag{5.30}$$

The chiral superfields Q contain the LH quark doublets, L the LH lepton doublets, and u^c, d^c, e^c the charge conjugates of the RH up–type quarks, RH down–type quarks and RH electron–type leptons respectively. Two Higgs fields are required to give masses separately to the up-type charge 2/3 quarks, and to the down-type charge-1/3 quarks and leptons; the last term is a mixing between them which is permitted by both gauge symmetry and supersymmetry (see § 5.3.2).

Supersymmetry must neccessarily be broken in the low energy world since the superpartners of the known particles (with the same mass) have not been observed.

However if supersymmetry is to provide a solution to the hierarchy problem, the mass-splitting $\tilde{m}$ between ordinary particles and their superpartners cannot be significantly higher than the electroweak scale. Although superparticles particles have not yet been directly produced at accelerators, they would influence through their virtual effects, the evolution with energy of the gauge couplings in the Standard Model. Interestingly enough, the precision data from *LEP* demonstrate that only in this case would there be the desired unification of all three couplings (e.g Ellis *et al* 1991, Amaldi *et al* 1991, Langacker and Luo 1991, see de Boer 1994); further this happens at a sufficiently high energy ($\approx 2 \times 10^{16}$ GeV), so as to account for the failure to detect proton decay upto a lifetime of $\sim 10^{32}$ yr (see Perkins 1984, Particle Data Group 1996) which rules out most non-SUSY GUTs (see Langacker 1981, Enqvist and Nanopoulos 1986). Reversing the argument, a SUSY-GUT can then predict, say the weak mixing angle, to a precision better than 0.1%, in excellent agreement with experiment (see Dimopoulos 1995, Ellis 1995). Another attraction of supersymmetry is that it provides a natural mechanism for breaking of the electroweak symmetry at the correct energy scale through radiative corrections to the Higgs mass, if the top quark is sufficiently heavy (see Ibáñez and Ross 1993); the recently discovered top quark does indeed have the required mass (Ross and Roberts 1992).

The first phenomenological models to be constructed attempted to incorporate global supersymmetry down to the electroweak scale (see Fayet and Ferrara 1975). Such models therefore contain a massless goldstino ($\tilde{G}$), the spin-$\frac{1}{2}$ fermion associated with the spontaneous breaking of supersymmetry at a scale of O(TeV), as well as a new light spin-$\frac{1}{2}$ fermion, the photino ($\tilde{\gamma}$). Both the goldstino and photino couple to matter with strength comparable to a doublet neutrino (see Fayet 1979). Thus, having two degrees of freedom each, they count as two extra neutrino species during nucleosynthesis and thus are in conflict with the often used bound $N_\nu < 4$ (Dimopoulos and Turner 1982), or even our conservative bound $N_\nu < 4.5$. Therefore it is neccessary to make these particles decouple earlier than the quark-hadron phase transition in order to dilute their abundance. To weaken their interactions adequately then requires that the supersymmetry breaking scale be raised above ≈ 10 TeV (Sciama 1982)

However models with global supersymmetry have severe difficulties, e.g. in generating the neccessary mass-splitting of $\mathcal{O}(m_W)$ between ordinary particles and their superpartners, and because they possess a large cosmological constant which cannot even be fine tuned to zero (see Fayet 1984). Thus it is neccessary to consider *local* supersymmetry, i.e. supergravity (see Van Nieuwenhuizen 1981), which provides an implicit link with gravity. (Indeed superstring theories (see Green *et al* 1993) which unify gravity with the other interactions, albeit in a higher-dimensional space, yield supergravity as the effective field theory in four-dimensions at energies small compared to the compactification scale which is of order $M_{\rm P}$.) In supergravity models, the goldstino is eliminated by the super-Higgs mechanism which gives a mass to the gravitino, spin-$\frac{3}{2}$ superpartner of the graviton (Deser and Zumino 1977). The helicity-$\frac{3}{2}$ components interact only gravitationally; however if the gravitino mass is very small then its interactions are governed by its helicity-$\frac{1}{2}$ component which is just the goldstino associated with global SUSY breaking (Fayet 1979). At energies much smaller than the superpartner masses, the typical scattering cross-section is $\sigma_{\tilde{G}\mathrm{e}\to\tilde{\gamma}\mathrm{e}} \approx 0.4\sigma_{\nu\mathrm{e}\to\nu\mathrm{e}}(m_{3/2}/10^{-5}\,\mathrm{eV})^{-2}$ (Fayet 1979), assuming that the photino is also light. Hence a sufficiently light gravitino would have the same cosmological abundance as a massless two-component neutrino. Requiring that the gravitino decouple at a temperature $T > m_\mu$ (see (4.4)) then implies (Fayet 1982)

$$m_{3/2} \gtrsim 10^{-2} \ \mathrm{eV} , \tag{5.31}$$

which is rather more restrictive than the lower limit of $\sim 10^{-6}\,\mathrm{eV}$ deduced from laboratory experiments (see Fayet 1987).

In fact the gravitino is expected to be much heavier in the class of supergravity models which have been phenomenologically most successful (see Nath *et al* 1984, Nilles 1984). Here supersymmetry is broken by non-perturbative dynamics at a scale Λ in a 'hidden sector' which interacts with the visible sector only through gravitational interactions (Witten 1981b, see Nilles 1990).† Supersymmetry breaking is then communicated to the low energy world only through 'soft' supersymmetry breaking masses for the sfermions and gauginos (superpartners of the fermions and gauge bosons) and a mass for the gravitino, all of which are of order the effective supersymmetry breaking scale in the visible sector, i.e. close to the electroweak scale (e.g. Barbieri *et al* 1982, Chamseddine *et al* 1982, Nilles *et al* 1983, Alvarez-Gaume *et al* 1983). Thus if supersymmetry is to solve the gauge hierarchy problem, the gravitino mass must be no higher than $\sim 1\,\mathrm{TeV}$. This however poses a serious cosmological problem as we discuss below.

5.3.1. The gravitino problem, baryogenesis and inflation: At high energies, the dominant interactions of the gravitino with other particles and their superpartners at high energies come from its helicity-$\frac{3}{2}$ component (rather than its helicity-$\frac{1}{2}$ goldstino component). For example it can decay into a gauge boson A_μ and its gaugino partner λ through a dimension-5 operator, with lifetime $\tau_{3/2\to A_\mu\lambda} \approx 4M_\mathrm{P}^2/N_\mathrm{c}m_{3/2}^3$ where N_c is the number of available channels, e.g.

$$\tau_{3/2\to\tilde{\gamma}\gamma} = 3.9 \times 10^5\,\mathrm{sec} \left(\frac{m_{3/2}}{\mathrm{TeV}}\right)^{-3} ,$$

† Alternatively, the 'messenger sector' can have gauge interactions so that the soft masses are generated by radiative corrections while the gravitino, which interacts only gravitationally, remains light: $m_{3/2} \sim m_W^2/M_\mathrm{P} \sim 10^{-6}\,\mathrm{eV}$ (e.g. Dine and Nelson 1993, Dine *et al* 1996).

$$\tau_{3/2\to\tilde{g}g} = 4.4\times10^4\,\mathrm{sec}\left(\frac{m_{3/2}}{\mathrm{TeV}}\right)^{-3}\ , \tag{5.32}$$

assuming $m_{A_\mu,\lambda} \ll m_{3/2}$.

It was first noted by Weinberg (1982) that notwithstanding their very weak interactions, massive gravitinos would have been abundantly produced in the early universe at temperatures close to the Planck scale and would thus come to matter-dominate the universe when the temperature dropped below their mass. Their subsequent decays would then completely disrupt primordial nucleosynthesis, thus creating a cosmological crisis for supergravity. It was suggested (Ellis *et al* 1983, Krauss 1983a) that this problem could be solved by invoking an inflationary phase, just as for GUT monopoles. However, unlike the latter, gravitinos can be recreated by scattering processes during the inevitable reheating phase following inflation as well as (in a model-dependent manner) through direct decays of the scalar field driving inflation (Nanopoulos *et al* 1983). The gravitino abundance produced by $2 \to 2$ processes involving gauge bosons and gauginos during reheating was computed by Ellis *et al* (1984b) to be,

$$\frac{n_{3/2}}{n_\gamma} = 2.4\times10^{-13}\left(\frac{T_\mathrm{R}}{10^9\,\mathrm{GeV}}\right)\left[1 - 0.018\ln\left(\frac{T_\mathrm{R}}{10^9\,\mathrm{GeV}}\right)\right]\ , \tag{5.33}$$

at $T \ll m_\mathrm{e}$, where T_R is the maximum temperature reached during reheating. This is a conservative lower bound to the true abundance, for example Kawasaki and Moroi (1995a) estimate an abundance higher by a factor of 4 after including interaction terms between the gravitino and chiral multiplets. Recently Fischler (1994) has claimed that gravitinos can be brought into thermal equilibrium at temperatures well below the Planck scale via interactions of their goldstino component with a cross-section which increases as T^2 due to the breaking of supersymmetry by finite temperature effects. If so, their production during reheating would be far more efficient and yield a relic abundance

$$\begin{aligned}\frac{n_{3/2}}{n_\gamma} &\approx \frac{g_*^{1/2}\alpha_\mathrm{s}^3 T^3}{m_{3/2}^3 M_\mathrm{P}^2}\\ &\sim 3\times10^{-13}\left(\frac{T_\mathrm{R}}{10^5\,\mathrm{GeV}}\right)^3\left(\frac{m_{3/2}}{\mathrm{TeV}}\right)^{-2}.\end{aligned} \tag{5.34}$$

However Leigh and Rattazzi (1995) argue on general grounds that there can be no such enhancement of gravitino production. Ellis *et al* (1996) perform an explicit calculation of finite-temperature effects for $m_{3/2}, \tilde{m} \ll T \ll \Lambda$ and demonstrate that these do not alter the estimate in (5.33).

The BBN constraints on massive decaying particles shown in figures 13 and 15 then provide a restrictive upper limit to the reheating temperature after inflation, dependent on the gravitino lifetime. For example, Ellis *et al* (1985b) quoted

$$T_{\rm R} \lesssim 2.5 \times 10^8\,{\rm GeV} \left(\frac{m_{3/2}}{100\,{\rm GeV}}\right)^{-1}, \quad {\rm for} \quad m_{3/2} \lesssim 1.6\,{\rm TeV}\ , \tag{5.35}$$

(taking $f_\gamma = 0.5$), from simple considerations of D + ^{3}He overproduction due to ^{4}He photofission which gave the constraint

$$m_{3/2}\frac{n_{3/2}}{n_\gamma} \lesssim 3 \times 10^{-12}\,{\rm GeV}\,f_\gamma^{-1}, \tag{5.36}$$

shown as a dashed line in figure 15. However, as is seen from the figure, a more detailed calculation of this process (Ellis *et al* 1992) actually yields a more restrictive constraint for a radiative lifetime greater than $\sim 2 \times 10^7$ sec, corresponding to $m_{3/2} \lesssim 300$ GeV, but a less stringent constraint for shorter lifetimes.†. Hence the true bound is (taking $f_\gamma = 0.5$),

$$T_{\rm R} \lesssim 10^8\,{\rm GeV} \quad {\rm for} \quad m_{3/2} = 100\ \ {\rm GeV}\,. \tag{5.37}$$

Using the results of Ellis *et al* (1992) we have obtained the upper bound on $T_{\rm R}$ implied by the relic abundance (5.33) as a function of the gravitino mass (calculated using the lifetime (5.32)) and show this in figure 17. For a gravitino mass of 1 TeV, the radiative lifetime is about 4×10^5 sec and the best constraint now comes from requiring that the photofission of deuterium not reduce its abundance below the observational lower limit (Juszkiewicz *et al* 1985, Dimopoulos *et al* 1989). The improved calculation of Ellis *et al* (1992) gives for this bound,

$$T_{\rm R} \lesssim 2.5 \times 10^9\,{\rm GeV} \quad {\rm for} \quad m_{3/2} = 1\ \ {\rm TeV}, \tag{5.38}$$

where we have taken $f_\gamma \simeq 0.8$ as is appropriate for such a massive gravitino. Photofission processes become ineffective for $\tau \lesssim 10^4$ sec but now there are new constraints from the effect of hadrons in the showers on the ^{4}He abundance (Reno and Seckel 1989, Dimopoulos *et al* 1989). If the gravitino mass is 10 TeV with a corresponding lifetime of $\tau_{3/2 \to \tilde{g}g} \sim 50$ sec, this bound is

$$T_{\rm R} \lesssim 6 \times 10^9\,{\rm GeV} \quad {\rm for} \quad m_{3/2} = 10\ \ {\rm TeV}\,. \tag{5.39}$$

Weinberg (1982) had suggested that the entropy release in the decays of a gravitino of mass exceeding ≈ 10 TeV would reheat the universe to a temperature high enough to restart nucleosynthesis, thus evading the cosmological problem. However, as noted earlier, particle decays following an exponential decay law cannot actually raise the

† Kawasaki and Moroi (1995a) quote a limit more stringent by a factor of about 100, of which, a factor of 4 comes from their more generous estimate of the relic gravitino abundance. The remaining discrepancy is because they obtain (by numerical integration of the governing equations) a significantly more stringent constraint on D + ^{3}He overproduction, which, as noted earlier, disagrees with both the analytic estimate of Ellis *et al* (1992) as well as the Monte Carlo calculation of Protheroe *et al* (1995).

temperature but only slow down its rate of decrease (Scherrer and Turner 1985), hence one should really require the gravitino to be massive enough that it decays before the begining of nucleosynthesis. A careful calculation by Scherrer *et al* (1991) taking into account the effects of hadronic decays shows that the lower bound on the mass is then

$$m_{3/2} \gtrsim 53 \ \text{TeV} . \tag{5.40}$$

However such a large gravitino mass cannot be accomodated in (minimal) supergravity models without destabilizing the hierarchy.

Other constraints on the gravitino abundance follow from examination of the effects of the annihilation of antiprotons produced in the decay chain $3/2 \to \tilde{g}g$, $\tilde{g} \to q\bar{q}\tilde{\gamma}$ (Khlopov and Linde 1984, Ellis *et al* 1985b, Halm 1987, Dominguez-Tenreiro 1987) but these are not as restrictive as those given above. The effects of the decay $3/2 \to \nu\tilde{\nu}$ have been studied by de Laix and Scherrer (1993). Correcting earlier estimates by Frieman and Giudice (1989) and Gratsias *et al* (1991), the tightest bound they obtain is $T_{\mathrm{R}} \lesssim 2 \times 10^{10}\,\text{GeV}$ for $m_{3/2} = 10\,\text{TeV}$. (Rather different bounds are obtained by Kawasaki and Moroi (1995c) by numerical solution of the governing equations but, as noted earlier, their cascade spectrum disagrees with that obtained by other workers.)

If the gravitino is in fact the LSP, then we can demand that its relic abundance respect the cosmological bound (4.25) on the present energy density in massive stable particles. Using (5.33), this requires (Ellis *et al* 1985b):

$$T_{\mathrm{R}} \lesssim 10^{12} \left(\frac{m_{3/2}}{100\,\text{GeV}}\right)^{-1} \ \text{GeV} , \tag{5.41}$$

while the decays of the next-to-lightest supersymmetric particle (NLSP), typically the neutralino χ^0, can presumably be made consistent with the BBN constraints since such particles can usually self-annihilate sufficiently strongly to reduce their relic abundance to an acceptable level. Moroi *et al* (1993) have reexamined this question and taken into account the (small) additional gravitino production from the NLSP decays. They note that the relic χ^0 abundance according to recent calculations (e.g. Drees and Nojiri 1993) is in fact sufficiently high (essentially due to improved lower limits on sparticle masses) that the $\mathrm{D} + {}^3\mathrm{He}$ photoproduction constraint calculated by Ellis *et al* (1992) requires $\tau_{\chi^0} \lesssim 5 \times 10^6\,\text{sec}$. For this to be so, the neutralino has to be sufficiently heavy relative to the gravitino, e.g for $m_{\chi^0} = 50\,\text{GeV}$, the gravitino mass must be less than $3.4\,\text{GeV}$, but for $m_{\chi^0} = 1\,\text{TeV}$ the gravitino mass can be as high as $772\,\text{GeV}$.

Moroi *et al* (1993) also reevaluate the bound (5.41) on T_{R} following from the relic energy density argument for a stable gravitino which is much lighter than the sfermions. They consider the regime $m_{3/2} \ll \tilde{m} \ll T$ for which they find that the gravitino (goldstino) annihilation cross-section is enhanced proportional to $m_{3/2}^{-2}$ so that the bound on T_{R} from consideration of the relic energy density decreases proportionally to $m_{3/2}$. Of course this bound evaporates when the gravitino becomes sufficiently light that its relic abundance from thermal equilibrium (at the Planck scale) comes within observational constraints. This limiting mass is (Pagels and Primack 1982)

$$m_{3/2} \lesssim 1 \text{ keV}; \tag{5.42}$$

the increase by a factor of $\approx$ 10 over the corresponding bound (5.1) for neutrinos is because of the dilution by a factor of $\approx$ 10 of gravitinos (which decouple at $T \gg T_{\rm c}^{\rm EW}$) relative to neutrinos which decouple at a few MeV (see table 1). Moroi *et al* also consider the effect of a light gravitino on the speed-up rate during BBN following Fayet (1982) (see (5.31)). They find that the relevant dominant thermalization process is gravitino (goldstino) annihilation to light lepton pairs and obtain the bound $m_{3/2} > 10^{-4}\,{\rm eV}(m_{\tilde{\ell}}/100\,{\rm GeV})$ by requiring decoupling at $T > T_{\rm c}^{\rm qh}$ (corresponding to the constraint $N_\nu < 3.3$). However Gherghetta (1996) notes that in the more appropriate regime $m_{3/2} \ll T \ll \tilde{m}$, the dominant equilibrating process is actually gravitino (goldstino) annihilation into photons, the cross-section for which is considerably smaller. Thus the mass bound is weakened to

$$m_{3/2} \gtrsim 10^{-6}\,{\rm eV}\left(\frac{m_{\tilde{\gamma}}}{100\,{\rm GeV}}\right)^{1/2}\ , \tag{5.43}$$

if the gravitinos are required to decouple at $T > m_\mu$, corresponding to the constraint $N_\nu < 3.7$ (compare with the bound (5.31) obtained assuming a light photino). Note that such a reduction of the annihilation cross-section would also degrade the bounds on $T_{\rm R}$ quoted by Moroi *et al* (1993).

The realization that the F-R-W universe we inhabit cannot have achieved a temperature higher than $\sim 10^9\,{\rm GeV}$, if the production of electroweak scale relic gravitinos is to be adequately suppressed, had a big impact on phenomenological models of baryogenesis. A decade ago when these bounds were first presented (e.g. Ellis *et al* 1985b) baryogenesis was generally believed to be due to the out-of-equilibrium B-violating decays of heavy bosons with masses of order the unification scale (see Kolb and Turner 1983). In supersymmetric models, protons can decay efficiently through dimension-5 operators (see Enqvist and Nanopoulos 1986), hence the experimental lower bounds $\tau_{{\rm p}\to\mu^+ K^0,\nu K^+} > 10^{32}\,{\rm yr}$ (Particle Data Goup 1996) then implies that the mass of the relevant (Higgs triplet) bosons is rather large, viz. $m_{\tilde{H}_3} \gtrsim 10^{16}\,{\rm GeV}$ (e.g. Ellis *et al* 1982). It may be possible to suppress the dangerous dimension-5 operators (e.g. Coughlan *et al* 1985) but even so one has a lower limit $m_{\tilde{H}_3} \gtrsim 10^{11}\,{\rm GeV}$ from consideration of the (unavoidable) dimension-6 operators . Such heavy particles *cannot* be thermally generated after inflation given the associated bounds on gravitino production, hence the standard scenario for baryogenesis is no longer viable!

This conflict motivated studies of various alternative possibilities. One suggestion was that the Higgs triplets could be created directly through the decays of the scalar field driving inflation (Coughlan *et al* 1985, Mahajan 1986). For this to be possible, the inflaton field should of course be significantly heavier than 10^{11} GeV. However the amplitude of the CMB temperature fluctuations observed by *COBE* , interpreted as due to quantum fluctuations of the inflaton field (see Liddle and Lyth 1993, Turner 1993), suggest an inflaton mass which is comparable in value (e.g. Ross and Sarkar 1996), thus leaving little room for manouver. A similar idea is to invoke decays of the inflaton into squarks, which decay in turn creating a baryon asymmetry if the superpotential includes a term which violates R-parity (Dimopoulos and Hall 1987). Here, the reheat temperature is required to be very low, less than a few GeV, in order that the generated asymmetry is not washed out by scattering processes and inverse decays. Another suggestion due to Fukugita and Yanagida (1986) which has received much attention is that the out-of-equilibrium L-violating decays of heavy right-handed neutrinos generate a *lepton* asymmetry, which is subsequently reprocessed by $B-L$ conserving fermion number violating interactions in the Standard Model (see below) into a baryon asymmetry. The $\nu_{\rm R}$ should thus have a mass less that $\sim 10^{11}$ GeV, which in turn imposes an interesting *lower* limit on the masses of the light (left-handed) neutrinos generated by the 'see-saw' mechanism, $m_{\nu_{\rm L}} \sim m_{q,l}^2/m_{\nu_{\rm R}}$ (see Plümacher 1996).

A different approach is to try and evade the gravitino problem altogether by decoupling the gravitino mass from the electroweak scale, i.e. making it lighter than 1 keV (5.42)), or heavier than 50 TeV (5.40). As mentioned earlier, this is not possible in minimal supergravity but can be achieved in 'no-scale' supergravity models which are based on non-compact Kähler manifolds (see Lahanas and Nanopoulos 1987).† Also the gravitino is naturally light in gauge-mediated supersymmetry breaking models (Dine and Nelson 1993, Dine *et al* 1996) which are consequently free of cosmological problems.

Thus the identification of the gravitino problem stimulated the study of mechanisms for low temperature baryogenesis. Coincidentally two such possibilities were proposed at that time and these have since come under intense scrutiny (see Dolgov 1992). The first followed the realization that since fermion number is violated (although $B-L$ is conserved) even in the Standard Model at temperatures above the electroweak scale (Kuzmin *et al* 1985), a baryon asymmetry may be generated utilising the (small) CP violation in the CKM mixing of the quarks if the neccessary out-of-equilibrium conditions can be achieved during the electroweak phase transition. (These detailed studies indicate however that successful baryogenesis probably requires extension of the SM to include, e.g. supersymmetry, in order to increase the sources of CP violation as well as make the electroweak phase transition sufficiently first-order (see Krauss and Rey 1992, Cohen *et al* 1994, Rubakov and Shaposhnikov 1996). The second mechanism,

† Here the scale of supersymmetry breaking (hence the gravitino mass) is classically undetermined and can be suitably fixed by radiative corrections. However, the construction of an acceptable cosmology is then beset by the 'Polonyi problem', viz. the late release of entropy stored in very weakly coupled fields in such models (e.g. German and Ross 1986, Ellis *et al* 1986c, Bertolami 1988).

specific to supergravity models, is based on the observation that sfermion fields will develop large expectation values along ‘flat’ directions in the scalar potential during the inflationary phase (Affleck and Dine 1984). A baryon asymmetry can then be generated at a much lower temperature, when the Hubble parameter becomes less than the effective mass, as the coherent oscillations in the fields decay (e.g. Ellis *et al* 1987). Recently, Dine *et al* (1995, 1996) have emphasized the role of non-renormalizable operators in the superpotential in stabilizing the flat direction and shown that a baryon asymmetry of the required magnitude can indeed arise.

In inflationary models, the reheat temperature $T_{\rm R}$ is determined by the couplings to matter fields of the scalar ‘inflaton’ field ϕ, which drives inflation at an energy scale Δ. Inflation ends when ϕ evolves to the minimum of its potential and begins to oscillate about it until it decays, converting its vacuum energy, Δ^4, into radiation. The inflaton is required to be a gauge singlet in order that its quantum fluctuations during the vacuum energy dominated ‘De Sitter phase’ of expansion do not generate temperature fluctuations in the CMB in excess of those observed by *COBE* (see Olive 1990a). Thus its couplings to matter fields are neccessarily very weak and reheating is a slow process. The inflaton mass is $m_\phi \sim \Delta^2/M_{\rm P}$ and its decay width is

$$\Gamma_\phi \sim \frac{m_\phi^3}{M_{\rm P}^2} \,, \tag{5.44}$$

so the maximum temperature reached during reheating is†

$$T_{\rm R} \sim (\Gamma_\phi M_{\rm P})^{1/2} \sim \frac{\Delta^3}{M_{\rm P}^2} \,. \tag{5.45}$$

The requirement that this be less than the phenomenological bound imposed by the gravitino problem poses a serious challenge for inflationary models (see Binétruy 1985, Enqvist 1986, Olive 1990a). As mentioned earlier the quantum fluctuations of the inflaton field generates density perturbations as it ‘rolls’ down its potential (see Brandenberger 1985) and these induce temperature fluctuations in the CMB. The *COBE* measurement of the CMB quadrupole anisotropy then fixes the amplitude of these perturbations on spatial scales of order the present Hubble radius and imposes an independent constraint on the ratio of the vacuum energy to the slope of the inflaton scalar potential. Since the vacuum energy is already bounded by the reheat constraint, this translates into an upper bound on the slope of the potential at the point where these fluctuations are generated, which can be identified by solving the equation of motion for the inflaton. Ross and Sarkar (1996) have shown that when the scalar potential is given by minimal $N = 1$ supergravity (Cremmer *et al* 1979, 1983), this constraint is *not*

† Recently, several authors (Kofman *et al* 1996, see Boyanovsky *et al* 1996) have reexamined the dynamics of reheating and identified mechanisms, e.g. parametric resonance, which can drastically change the simple picture discussed here, for specific couplings of the inflaton to matter fields.

satisfied by generic 'chaotic' inflationary models (see Linde 1990) wherein the inflaton evolves towards its global minimum at the origin from an initial VEV beyond the Planck scale. However it is easy to satisfy this constraint in a 'new' inflationary model (Holman *et al* 1984) where the inflaton evolves from an initial value at the origin towards its minimum at the Planck scale. The scalar potential is then as flat as is required, with $\Delta \sim 3 \times 10^{14}\,\mathrm{GeV}$, i.e. m_ϕ of $\mathrm{O}(10^{11})\,\mathrm{GeV}$ as mentioned earlier. Two observationally testable consequences of this model are that gravitational waves contribute negligibly to the CMB anisotropy and the spectrum of scalar density perturbations is 'tilted' with a slope of about 0.9, which improves the fit to the observed clustering and motions of galaxies in an universe dominated by cold dark matter (Sarkar 1996, Adams *et al* 1996).

A related issue is the aforementioned Polonyi problem (Coughlan *et al* 1983, 1984, Dine *et al* 1984, Goncharov *et al* 1984) associated with very weakly coupled light scalar fields which are driven out to large VEVs along flat directions during the De Sitter phase (if the Hubble parameter exceeds their mass). Subsequently the field evolves towards its minimum just like the inflaton field and eventually reaches the minimum and oscillates about it converting the stored vacuum energy into radiation. However this happens very late for a light singlet field (see (5.44)) hence the universe reheats to a temperature *below* the value of $\mathrm{O}(10)\,\mathrm{MeV}$ required for starting off successful nucleosynthesis. Hence it is essential to address this problem which is particularly acute in models derived from (compactified) string theories, because of the associated 'moduli' fields which do exhibit such flat directions and have masses at most of order the electroweak scale, giving a reheat temperature of

$$T_{\mathrm{R}} \sim m_\phi^{3/2} M_{\mathrm{P}}^{-1/2} \sim 10^{-6}\,\mathrm{GeV} \tag{5.46}$$

(e.g. de Carlos *et al* 1993, Banks *et al* 1994). One way to evade this is to invoke a second phase of inflation with a Hubble parameter *smaller* than the moduli mass, with a reheat temperature high enough not to disturb nucleosynthesis (Randall and Thomas 1995); the second epoch of inflation must of course be short enough not to erase the density perturbations produced in the initial inflationary era but long enough to solve the moduli problem. A natural mechanism for such 'thermal inflation' thas recently been identified in models with an intermediate symetry breaking scale (Lyth and Stewart 1995, 1996). There are other possible solutions to the moduli problem (e.g. Dine *et al* 1995, 1996, Thomas 1995, Banks *et al* 1995a,b, Ross and Sarkar 1996, Linde 1996) — for example all moduli may have VEVs fixed by a stage of symmetry breaking *before* inflation, or the moduli minima may be the *same* during and after inflation, corresponding to a point of enhanced symmetry. In the first case, the 'dilaton' field, which determines the string coupling, should also acquire a (non-perturbative) mass much higher than the electroweak scale since otherwise the curvature of the potential in the dilaton direction is too great to allow for a period of inflation (Brustein and

Steinhardt 1993). In both cases the implication is that the moduli *cannot* be treated (cf. Kounnas *et al* 1994, Binétruy and Dudas 1995, Dimopoulos *et al* 1995) as dynamical variables at the electroweak scale, determining the couplings in the low energy theory. All this is a direct consequence of the requirement that standard BBN not be disrupted by the late release of entropy!

5.3.2. The 'μ–problem' and the NMSSM: As we have seen, the major motivation for adding (softly broken) supersymmetry to the Standard Model is to bring under control the quadratic divergences associated with a fundamental Higgs boson and make it 'natural' for its mass to be at the electroweak scale. However the minimal version of the supersymmetric Standard Model has a new naturalness problem associated with the mixing term $\mu\, H_1 H_2$ between the two Higgs doublets (see (5.30)). For successful phenomenology μ should also be of order the electroweak scale but there is no symmetry which ensures this — the 'μ–problem' (e.g. Hall *et al* 1983, Kim and Nilles 1984). To address this, the MSSM is extended by the addition of a singlet Higgs superfield N in the next-to-minimal supersymmetric Standard Model (NMSSM) (e.g. Nilles *et al* 1983, Derendinger and Savoy 1984). By invoking a Z_3 symmetry under which every chiral superfield Φ transforms as $\Phi \to \mathrm{e}^{2\pi i/3}\Phi$, the allowed terms in the superpotential are

$$P_{\mathrm{NMSSM}} = h_u Q H_2 u^c + h_d Q H_1 d^c + h_e L H_1 e^c + \lambda N H_1 H_2 - \frac{1}{3} k N^3 \ , \quad (5.47)$$

while the Higgs part of the soft supersymmetry breaking potential is extended by the inclusion of two additional trilinear soft terms A_λ and A_k to

$$\begin{aligned} V_{\mathrm{soft}}^{\mathrm{Higgs}} = & -\lambda A_\lambda (N H_1 H_2 + \mathrm{h.c.}) - \frac{1}{3} k\, A_k (N^3 + \mathrm{h.c.}) \\ & + m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 + m_N^2 |N|^2, \end{aligned} \quad (5.48)$$

where $H_1 H_2 = H_1^0 H_2^0 - H^- H^+$. The μ–term can now be simply set to zero by invoking the Z_3 symmetry. An effective μ–term of the form $\lambda\langle N\rangle$ will still be generated during $SU(2)_{\mathrm{L}} \otimes U(1)_Y$ breaking but it is straightforward to arrange that $\langle N\rangle$ is of order a soft supersymmetry breaking mass. Apart from solving the 'μ–problem' the NMSSM has interesting implications for supersymmetric Higgs phenomenology (e.g. Ellis *et al* 1989, Elliot *et al* 1994) and dark matter (e.g. Olive and Thomas 1991, Abel *et al* 1993).

Unfortunately, the NMSSM has a cosmological problem. The Z_3 of the model is broken during the phase transition associated with electroweak symmetry breaking in the early universe. Due to the existence of causal horizons in an evolving universe, such spontaneously broken discrete symmetries lead to the formation of domains of different degenerate vacua separated by domain walls (Zel'dovich, Kobzarev and Okun 1975, Kibble 1976). These have a surface energy density $\sigma \sim \nu^3$, where ν is a typical VEV of the fields, here of order the electroweak scale. Such walls would come to dominate the energy density of the universe and create unacceptably large anisotropies in the CMB unless their energy scale is less than a few MeV (see Vilenkin 1985). Therefore cosmology requires electroweak scale walls to disappear well before the present era. Following the original suggestion by Zel'dovich *et al* (1975), this may be achieved by breaking the degeneracy of the vacua, eventually leading to the dominance of the true vacuum. This happens when the pressure ε, i.e. the difference in energy density between the distinct vacua, begins to exceed the tension σ/R, where σ is the surface energy density of the walls and R the scale of their curvature. When R becomes large enough for the pressure term to dominate, the domain corresponding to the true vacuum begins to expand into the domains of false vacuum and eventually fills all of space. It has been argued (Ellis *et al* 1986a, Rai and Senjanović 1994) that strong gravitational interactions at the Planck scale, which are expected to explicitly violate any discrete symmetry, would cause just such a non-degeneracy in the minima of $\mathrm{O}(\nu^5/M_\mathrm{P})$ where ν is a generic VEV, of $\mathrm{O}(m_W)$ in the present case.

Abel *et al* (1995) have tested whether the above solution is indeed viable by studying the cosmological evolution of the walls under the influence of the tension, the pressure due to the small explicit Z_3 breaking and the friction due to particle reflections (see Vilenkin and Shellard 1994). They find that in order to prevent wall domination of the energy density of the universe one requires $\varepsilon > \sigma^2/M_\mathrm{P}^2$, a pressure which can be produced by dimension-6 operators in the potential. A much tighter constraint however comes from requiring that primordial nucleosynthesis not be disrupted by the decays of the walls into quarks and leptons. The energy density released in such collisions at time t is

$$\frac{\rho_\mathrm{walls}}{n_\gamma} \sim \frac{\sigma}{t n_\gamma} \approx 7\times 10^{-11}\,\mathrm{GeV}\left(\frac{\sigma}{m_W^3}\right)\left(\frac{t}{\mathrm{sec}}\right)^{1/2} . \tag{5.49}$$

The walls must disappear before $t \approx 0.1\,\mathrm{sec}$ to ensure that the hadrons produced in their decays do not alter the neutron-to-proton ratio, resulting in a ^{4}He mass fraction in excess of 25% (see figure 15). This requires the magnitude of explicit Z_3 breaking to be

$$\varepsilon \gtrsim \lambda' \frac{\sigma m_W^2}{M_\mathrm{P}} , \tag{5.50}$$

with $\lambda' \sim 10^{-7}$. Thus addition of a dimension-5 non-renormalizable operator to the superpotential is sufficient to evade the cosmological constraints. However this creates a naturalness problem since introduction of non-renormalizable terms together with soft supersymmetry breaking produces corrections to the potential which are quadratically divergent and thus proportional to powers of the cut-off Λ in the effective supergravity theory (e.g. Ellwanger 1983, Bagger and Poppitz 1993). Since the natural scale for this cut-off is M_P, these can destabilize the hierarchy, forcing the singlet VEV (and hence the scale of electroweak symmetry breaking) upto, at least, the hidden sector scale of $\approx (m_{3/2}M_\mathrm{P})^{1/2} \sim 10^{11}\,\mathrm{GeV}$. By examining the possible dimension-5 Z_3 breaking terms, Abel *et al* (1995) demonstrate that this can be averted only if the coefficient λ' in (5.50) is smaller than 3×10^{-11}. Thus the NMSSM has either a cosmological domain wall problem or a hierarchy problem.

A possible solution is to reintroduce the μ term in the superpotential in such a way as to avoid the introduction of the dangerous non-renormalizable operators. By allowing specific couplings of the hidden sector fields to the visible sector (Giudice and Masiero 1988), one can retain Z_3 symmetry in the full theory but break it spontaneously when supersymmetry is broken; then the hierarchy is not destabilized by tadpole diagrams. Nevetheless allowed operators which would give N a mass of order the SUSY breaking scale still have to be set to zero by hand and this constitutes a naturalness problem of at least one part in 10^9 (Abel *et al* 1995).

5.3.3. R-parity breaking and LSP decays: Apart from the terms in the MSSM superpotential shown in (5.30), one can in general add other gauge-invariant terms such as (Hall and Suzuki 1984)

$$P_{\not R} = \lambda_{ijk} L_i L_j e^c_k + \lambda'_{ijk} L_i Q_j d^c_k + \lambda''_{ijk} u^c_i d^c_j d^c_k \ , \tag{5.51}$$

where L_i and Q_i are the $SU(2)$-doublet lepton and quark superfields and e^c_i, u^c_i, d^c_i are the singlet superfields. These are phenomenologically dangerous since the λ and λ' couplings violate lepton number, while λ'' couplings violate baryon number. Hence such couplings are usually eliminated by enforcing a discrete symmetry termed R–parity, $R \equiv (-1)^{3B+L+2S}$ (Farrar and Fayet 1978). This has the additional important consequence that the lightest superpartner (LSP) is absolutely stable and therefore a good dark matter candidate. However, an exact R-parity is not essential from a theoretical point of view, since rapid proton decay can be prevented by simply requiring that all the λ''_{ijk} in (5.51) be zero as a consequence of an underlying symmetry.† It has been argued (Campbell *et al* 1991, Fischler *et al* 1991) that the other terms must also be zero or very small in order for a primordial baryon asymmetry to survive since this requires that B and/or L-violating interactions should not have come into thermal equilibrium above the electroweak scale when ($B - L$ conserving) fermion number violation is already unsuppressed in the Standard Model (Kuzmin *et al* 1985). However this can be evaded, for example, through lepton mass effects which allow a baryon asymmetry to be regenerated at the electroweak scale through sphaleron processes if there is a primordial flavour-dependent lepton asymmetry (Kuzmin *et al* 1987, Dreiner

† R-parity may also be broken spontaneously if a neutral scalar field, viz. the sneutrino, gets a VEV (Aulakh and Mohapatra 1982) but this gives rise to a problematical massless Goldstone boson, the Majoron, unless explicit $\not R_{\rm p}$ is also introduced in some way. Moreover, this induces a neutrino mass and the scenario is thus constrained by the BBN bounds (§ 5.1.2) on an unstable ν_τ (e.g. Ellis *et al* 1985a).

and Ross 1993).‡ There are also other possibilities for protecting a baryon asymmetry (e.g. Campbell *et al* 1992a, Cline *et al* 1994) so this is not a firm constraint.

The cosmological consequences of R-parity violation have been examined by Bouquet and Salati (1987). The LSP, which is usually the neutralino, is now unstable against tree-level decays (similar to a heavy neutrino) with lifetime

$$\tau \approx \frac{10^{-16}\,\mathrm{sec}}{\lambda^2}\left(\frac{m_{\chi^0}}{10\,\mathrm{GeV}}\right)^{-5}\left(\frac{m_{\tilde{f}}}{100\,\mathrm{GeV}}\right)^{4} \quad . \tag{5.52}$$

where $m_{\tilde{f}}$ is the mass of the squark/slepton as appropriate to the $R\!\!\!/_{\mathrm{p}}$ coupling λ under consideration. Such decays must of course occur early enough so as not to disturb BBN, hence the arguments of § 4.2 impose a *lower* bound on the $R\!\!\!/_{\mathrm{p}}$ coupling. The precise bound on the lifetime depends on the decay mode, e.g. whether or not the final state includes hadrons. The relic abundance of a LSP heavier than a few MeV freezes-out before BBN, therefore the usual calculation (e.g. Ellis *et al* 1984a) can be used, ignoring the effect of LSP decays. For example, a neutralino of mass 10 GeV has a relic abundance of $n_{\chi^0}/n_\gamma \sim 3\times10^{-8}\,\mathrm{GeV}\,(m_{\tilde{f}}/100\,\mathrm{GeV})^4$; if it decays through the operators λ'_{LQd} or λ''_{udd} creating hadronic showers, then figure 15 shows that we can require $\tau_{\chi^0} \lesssim 1\,\mathrm{sec}$, i.e.

$$\lambda \gtrsim 10^{-8}. \tag{5.53}$$

(Laboratory experiments looking for $R\!\!\!/_{\mathrm{p}}$ effects are sensitive to a *maximum* lifetime of $\mathrm{O}(10^{-6})$ sec so can only probe $\lambda \gtrsim 10^{-5}$.) Of course λ may be very small (or indeed zero!) making the neutralino lifetime longer than the age of the universe. Then the BBN constraints do not apply but arguments based on the absence of a high energy neutrino background require the lifetime to be higher than $\approx 3\times10^{17}$ yr (Gondolo *et al* 1993), thus implying $\lambda \lesssim 10^{-21}$ (e.g. Campbell *et al* 1992b). Similar arguments have been used to constrain the destabilization of the LSP through $R\!\!\!/_{\mathrm{p}}$ in the singlet sector of the NMSSM (Allahverdi *et al* 1994).

5.3.4. Superstring models and new gauge bosons: Phenomenological models "motivated" by the superstring often contain additional neutral particles in each fermion generation, notably right-handed neutrinos which are singlets of the Standard Model (see Ellis 1987, Hewett and Rizzo 1989). These are often massless or very light and

‡ There is however no such loophole for $\Delta B = 2$, $\Delta L = 0$ interactions such as the dim-9 operator $(qqqqqq)/M^5$ (usually heavy Higgs exchange in unified theories) which mediate neutron-antineutron oscillations (Mohapatra and Marshak 1980). Such processes involve only quark fields, hence no flavour symmetry can be separately conserved because of CKM mixing. The experimental lower limit $\tau_{n-\bar{n}} \gtrsim 10^8\,\mathrm{sec}$ (Particle Data Group 1996) implies that such oscillations can have no influence on nucleosynthesis (Sarkar 1988). Demanding that such processes do not come into thermal equilibrium in the early universe requires $M \gtrsim 10^{14}\,\mathrm{GeV}$ (Campbell *et al* 1991, 1992b).

thus relativistic at the time of nucleosynthesis. However they couple to matter not through the Z^0 but through a hypothetical new neutral gauge boson Z' which is experimentally required to be heavier than the Z^0. Thus the $\nu_{\rm R}$ energy density at the time of nucleosynthesis will be suppressed relative to the conventional $\nu_{\rm L}$ if its interactions are sufficiently weak (i.e. the Z' is sufficiently heavy) to move the $\nu_{\rm R}$ decoupling back earlier than some epoch of entropy generation, e.g. $\mu^+\mu^-$ annihilation or the quark-hadron phase transition. Then each $\nu_{\rm R}$ will only count as a fraction of a $\nu_{\rm L}$ and possibly satisfy the BBN bound on N_ν, which thus translates into a lower bound on the mass of the Z'. Ellis *et al* (1986b) argued for the conservative constraint $N_\nu \lesssim 5.5$ (4.6) and noted that this would permit one additional $\nu_{\rm R}$ per generation if these decoupled before $\mu^+\mu^-$ annihilation thus ensuring $T_{\nu_{\rm R}}/T_{\nu_{\rm L}} < 0.59$. As discussed in § 4.1, this requires $\alpha \equiv \langle\sigma v\rangle_{\ell^+\ell^-\to\nu_{\rm R}\bar{\nu_{\rm R}}} < 1.4\times10^{-15}{\rm GeV}^{-4}$, but a more careful analysis of decoupling yields a bound *less* stringent than this naïve estimate (Ellis *et al* 1986b)

$$\langle\sigma v\rangle_{\ell^+\ell^-\to\nu_{\rm R}\bar{\nu_{\rm R}}} < 7\times10^{-15}T^2\ {\rm GeV}^4\ . \tag{5.54}$$

For the $SU(3)_{\rm c}\otimes SU(2)_{\rm L}\otimes U(1)_Y\otimes U(1)_{\rm E}$ model obtained from Calabi-Yau compactification (Witten 1985b, Dine *et al* 1985), the couplings of the new gauge boson Z_η are specified, e.g. $g_\eta = e/\cos\theta_{\rm W}$, enabling the annihilation cross-section above to be computed (Ellis *et al* 1986b). The above argument then gives $m_{Z'} \gtrsim 330\,{\rm GeV}$, which has only recently been matched by direct experimental bounds on such a new gauge boson (Particle Data Group 1996). Of course the cosmological bound would be more stringent still if one were to adopt a more restrictive constraint, e.g. taking $N_\nu \lesssim 4$ would require $m_{Z'} \gtrsim 780\,{\rm GeV}$ (Ellis *et al* 1986b). Even more restrictive bounds have been quoted (Steigman *et al* 1986, Gonzalez-Garcia and Valle 1990, Lopez and Nanopoulos 1990, Faraggi and Nanopoulos 1991) but these were based on an approximate treatment of decoupling and adopted more restrictive, but less reliable, constraints from BBN.

5.3.5. Supersymmetry breaking: Perhaps the most crucial issue in phenomenological supergravity concerns how supersymmetry is actually broken. As mentioned earlier, the most popular option is to break supersymmetry non-perturbatively in a hidden sector which interacts with the visible sector only through gravitational interactions (see Amati *et al* 1988, Nilles 1990) or through gauge interactions (Dine and Nelson 1993). A spontaneously broken R-symmetry is neccessary and sufficient for such dynamical supersymmetry breaking (Affleck *et al* 1985, Nelson and Seiberg 1994) and implies the existence of a pseudo Nambu-Goldstone boson, the R-axion. In renormalizable hidden sector models, the axion has a decay constant of ${\rm O}(M_{\rm SUSY})$ and a mass of ${\rm O}(m_{3/2}^{1/2}M_{\rm SUSY}^{1/2}) \sim 10^7\,{\rm GeV}$, where

$$M_{\rm SUSY} \sim (m_{3/2}M_{\rm P})^{1/2} \sim 10^{11}\,{\rm GeV} \tag{5.55}$$

is the effective scale of SUSY breaking in the hidden sector. The axion field is set oscillating during inflation and the energy density contained in such oscillations after reheating is released as the axions decay into both visible particles and gravitinos. As discussed by Bagger *et al* (1994) this is constrained by the bounds on massive decaying particles discussed in § 4.2. The implied upper limit on the reheat temperature is found to be competetive with that obtained (§ 5.3.1) from considerations of thermal gravitino generation. (Bagger *et al* (1994) also note that in visible sector models wherein SUSY breaking is communicated through gauge interactions, it is neccessary to have $M_{\rm SUSY}$ higher than $\sim 10^5\,{\rm GeV}$ in order to make the axion heavier than $\sim 100\,{\rm MeV}$ so as to evade astrophysical bounds (see Raffelt 1990).)

In non-renormalizable hidden sector models, the scale of gaugino condensation in the hidden sector due to Planck scale interactions is

$$\Lambda \sim M_{\rm SUSY}^{2/3} M_{\rm P}^{1/3} \ , \tag{5.56}$$

and the R-axion mass is of $\mathrm{O}(M_{\rm SUSY}^2/M_{\rm P}) \sim 10^3\,{\rm GeV}$ while its decay constant is of $\mathrm{O}(M_{\rm P})$. Banks *et al* (1994) note that this will give rise to a Polonyi problem and conclude that all such models are thus ruled out. However Rangarajan (1995a) has specifically considered the $E_8 \otimes E_8'$ superstring model compactified on a Calabi-Yau manifold (Gross *et al* 1985) and calculated the energy density in the coherent oscillations of the axions given their low temperature potential (e.g. Choi and Kim 1985). He finds that the axions decay before nucleosynthesis as required (§ 4.2) if

$$\Lambda \gtrsim 10^{13}\,{\rm GeV} \ , \tag{5.57}$$

consistent with the expected value of $\Lambda \simeq 5 \times 10^{13}\,{\rm GeV}$ (Derendinger *et al* 1985). The decay of the axion oscillations does increase the comoving entropy by a factor of $\sim 10^7$, but this is deemed acceptable if baryogenesis occurs by the Affleck-Dine mechanism (Rangarajan 1995b).

Yet another application of BBN bounds has been to orbifold compactifications of the superstring wherein supersymmetry is broken *perturbatively* by the Scherk-Schwarz mechanism at the electroweak scale (e.g. Rohm 1984, Antoniadis *et al* 1988, Ferrara *et al* 1989) by postulating the existence of a large internal dimension (see Antoniadis 1991). Thus, in addition to the MSSM, these models contain a repeating spectrum of Kaluza-Klein (KK) modes all the way up to the Planck scale, whose spacing ($\epsilon \approx 1/2R$, where R is the radius of compactification) is comparable to the supersymmetry breaking scale of $\mathrm{O}(\mathrm{TeV})$. Such modes can be directly excited at forthcoming accelerators such as the LHC (Antoniadis *et al* 1994), hence this possibility is of great experimental interest. These modes will also be excited in the early universe and this radically alters the thermal history (Abel and Sarkar 1995). The KK modes are labelled by quantum numbers of internal momenta/charges which are of the form

$$P_{\rm LR} = \frac{n}{R} \pm \frac{mR}{2} \ , \tag{5.58}$$

where R represents some internal radius of compactification. The winding modes ($m \neq 0$) have masses of $\mathrm{O}(M_{\rm P})$ and need not be considered further, while the particles in the nth KK mode have masses $m_n \sim n\epsilon$. Thus above the compactification scale, when the temperature rises by ϵ, two new levels of (gauge interacting) KK excitations become relativistic, so that the number of relativistic degrees of freedom increases *linearly* with temperature. For example, the number of entropic degrees of freedom rises above the limiting value $\hat{g}_s = 915/4$ (2.74) in the MSSM according to

$$g_s(T) = \hat{g}_s + \frac{T}{\epsilon} g_{s\,\rm KK} \ , \tag{5.59}$$

where the constant $g_{s\,\rm KK}$ is determined by evaluating the entropy density of the plasma. (In the spontaneously broken string theories, each KK level comes in $N = 4$ multiplets, so that KK gauge bosons contribute 8 bosonic and 8 fermionic degrees of freedom in the vector and fermionic representations of SO(8) respectively; in the minimal case in which the KK excitations are in $SU(3) \otimes SU(3)_{\rm c}$ multiplets, this gives e.g. $g_{s\,\rm KK} = 1400$.) Thus at a temperature much higher than the KK level-spacing ($T \gg \epsilon$), nearly all the entropy is contained in the KK modes and almost none in the matter multiplets. By entropy conservation, the Hubble expansion rate is then altered from its usual form (2.64) as

$$H = -\frac{4}{3}\frac{\dot{T}}{T} = 1.66 \sqrt{\frac{g_{s\,\rm KK}}{\epsilon}}\ \frac{T^{5/2}}{M_{\rm P}} \ . \tag{5.60}$$

Now consider the history of the universe starting from the maximum temperature it reached, viz. the reheating temperature $T_{\rm R}$ ($\gg \epsilon$) at the end of inflation. The entropy is initially evenly spread out amongst the strongly (as opposed to gravitationally) interacting KK modes and the massless matter multiplets. Until the temperature drops below the first KK level, the evolution of the universe is therefore governed by the KK modes, whose contribution to the entropy is continually decreasing as the temperature drops. During this period there is production of massive gravitons and gravitinos which can only decay to the massless (twisted) particles since their decays to the (untwisted) KK modes is kinematically suppressed. The effect of the decaying particles on the abundances of the light elements then imposes a severe bound on $T_{\rm R}$ (Abel and Sarkar 1995). For (hadronic) decays occuring before the begining of nucleosynthesis, the requirement that $Y_{\rm p}(^4{\rm He})$ not be increased above 25% translates into the bound

$$T_{\rm R} \lesssim 2 \times 10^4\,{\rm GeV} \left(\frac{\epsilon}{\rm TeV}\right)^{1/3} , \tag{5.61}$$

while for later decays, consideration of ^{2}H photofission imposes an even stricter bound. However the reheat temperature expected in these models is expected to be significantly larger than the usual value (5.45), viz.

$$T_{\rm R} \sim \left(\frac{g_{s\,\rm KK}}{\epsilon}\right)^{-1/4} (\Gamma_\phi M_{\rm P})^{1/2} \sim 10^6 \;\; {\rm GeV} \;\; \left(\frac{\epsilon}{\rm TeV}\right)^{1/4} , \qquad (5.62)$$

for an inflaton mass $m_\phi \sim 10^{11}$ GeV as is required to reproduce the *COBE* measurement of CMB fluctuations. A possible solution would appear to be a second phase of inflation with $m_\phi \sim \epsilon$ to dilute the KK states but the reheat temperature is then of $\mathrm{O}(10^{-6})$ GeV i.e. too low for nucleosynthesis to occur. Thus it appears to be difficult to construct a consistent cosmological history for four-dimensional superstring models with tree-level supersymmetry breaking, notwithstanding their many theoretical attractions.

5.4. Grand unification and cosmic strings

Phase transitions associated with the spontaneous breaking of a symmetry in the early universe can create stable topological defects in the associated Higgs field, viz. domain walls, strings and monopoles (Kibble 1976, see Vilenkin and Shellard 1994). Stable domain walls are cosmologically unacceptable, as we have seen earlier (§ 5.3.2), and so are monopoles, which are neccessarily created during GUT symmetry breaking (see Preskill 1984). Such monopoles would be expected to have a relic abundance comparable to that of baryons, but are $\sim 10^{16}$ times heavier, so would clearly lead to cosmological disaster. Further, direct searches have failed to find any monopoles (see Particle Data Group 1996). The most attractive mechanism for getting rid of then is to invoke an inflationary phase, with reheating to a temperature well below the GUT scale, as is also required independently from consideration of the gravitino problem (§ 5.3.1).

By contrast, cosmic strings have an acceptably small relic energy density and have been studied in great detail because they provide an alternative to inflationary scalar field fluctuations as the source of the perturbations which seed the growth of large-scale structure (see Brandenberger 1991, 1994). Detailed numerical studies (e.g. Albrecht and Stebbins 1992) find that this requires the string tension μ to be in the range

$$\mu \approx 1 - 4 \times 10^{-6} M_{\rm P}^2 , \qquad (5.63)$$

interestingly close to the GUT scale, and in agreement with the value $\mu = 2 \pm 0.5 \times 10^{-6} M_{\rm P}^2$ obtained (Coulson *et al* 1994, see also Bennett *et al* 1992) by normalizing the associated CMB fluctuations to the *COBE* data. An interesting constraint on such GUT scale strings follows from Big Bang nucleosynthesis (e.g. Hogan and Rees 1984, Brandenberger *et al* 1986, Bennett 1986, Quiròs 1991). The evolving network of cosmic strings generates gravitational radiation which contributes to the total relativistic energy density, thus the bound on the speed-up rate translates into an upper bound on the string tension. From a detailed numerical study, Caldwell and Allen (1992) find that the bound $N_\nu \leq 3.4$ (4.8) implies

$$\mu < 7 \times 10^{-6} M_{\rm P}^2 \ . \tag{5.64}$$

These authors illustrate how the bound is weakened if one adopts a more conservative bound, e.g. $N_\nu \leq 4$ implies $\mu < 1.6 \times 10^{-5} M_{\rm P}^2$. In general, consideration of the effect of the gravitational wave background on pulsar timing observations gives more stringent bounds (see Hindmarsh and Kibble 1995) but these too are consistent with the value (5.63) required for structure formation.

It has been noted that cosmic strings are likely to be superconducting so that large currents can be induced in them by a primordial magnetic field (Witten 1985a). In addition to gravitational waves, such a string also radiates electromagnetic radiation at an ever-increasing rate as its motion is damped thus increasing the current. Thus the end point is expected to be the catastrophic release of the entire energy content into high energy particles; such explosions will send shock waves into the surrounding intergalactic medium and galaxy formation may take place in the dense shells of swept-up matter (Ostriker, Thomson and Witten 1986). However this is constrained by the stringent bounds on such energy release during the nucleosynthesis era (Hodges and Turner 1988, Sigl *et al* 1995) and the idea is essentially ruled out by other constraints from the thermalization of the blackbody background (e.g. Wright *et al* 1994).

5.5. *Miscellaneous bounds*

There have been other applications of BBN constraints to hypothetical particles which do not fit into the categories considered above. For example bounds on scalars and pseudo-scalars (e.g. Bertolini and Steigman 1992) have been applied to hadronic axions (Chang and Choi 1993), to a particle which couples to two photons but not to leptons or quarks (Massó and Toldrà 1994b) and to Majoron emission in $\beta\beta$-decay (Chang and Choi 1994). Bounds have been derived on 'shadow matter' in superstring theories (Kolb *et al* 1985, Krauss *et al* 1986), on the time-evolution of possible new dimensions (Kolb *et al* 1986a, Barrow 1987) and on 'mirror fermions' (e.g. Fargion and Roos 1984, Senjanović 1986, Berezhiani and Mohapatra 1995, Foot and Volkas 1995b). Carlson and Glashow (1987) have ruled out a suggested solution to the orthopositronium decay puzzle involving 'milli-charged' particles (also discussed by Davidson and Peskin 1994) while Escribano *et al* (1995) have ruled out another solution involving exotic particle emission. For lack of space, we do not discuss these results except to caution that many of them assume an overly restrictive limit on N_ν and should be suitably rescaled to the conservative bound (4.13).

5.6. Implications for the dark matter

The nature of the dark matter which is observed to dominate the dynamics of individual galaxies as well as groups and clusters of galaxies (see Faber and Gallagher 1979, Trimble 1987, Ashman 1992) is one of the key problems at the interface of particle physics and cosmology. It may just be ordinary matter in some non-luminous form, e.g. planets, white dwarfs, black holes *et cetera* (see Lynden-Bell and Gilmore 1990, Carr 1994). However the BBN bound on the abundance of nucleons in *any* form constrains this possibility and implies that most of the dark matter is in fact non-nucleonic.

5.6.1. 'Baryonic' dark matter: The usually quoted BBN value of $\Omega_{\rm N} \approx 0.011h^{-2}$ (3.80) is significantly higher than the value (3.30) obtained from direct observations of luminous matter (3.30), as shown in figure 18. This suggests that most nucleons are dark and, in particular, that much of the dark matter in galactic halos, which contribute $\Omega \approx 0.05h^{-1}$ (see Binney and Tremaine 1987), may be nucleonic. However if the indications of a high primordial D abundance (Songaila *et al* 1994, Rugers and Hogan 1996a,b) are correct, then the implied lower value of $\Omega_{\rm N} \approx 0.0058\,h^{-2}$ (3.81) is close to its observational lower limit (for high values of h), leaving little room for nucleonic dark matter. Conversely, if the primordial D abundance is as low as found by Tytler *et al* 1996, the corresponding value of $\Omega_{\rm N} \approx 0.023\,h^{-2}$ (3.82) would suggest that much of the halo dark matter is nucleonic. Either possibility is consistent with searches to date for gravitational microlensing events expected (Paczyński 1986) for a halo dominated by dark compact objects. The 8 candidate events detected by the *MACHO* collaboration imply that about half of the halo mass can be in the form of such objects having a most probable mass of $0.5^{+0.3}_{-0.2}M_\odot$ (Alcock *et al* 1995, 1996).

Even more interesting is the comparison with clusters of galaxies which clearly have a large nucleonic content, particularly in the form of X-ray emitting intracluster gas. White *et al* (1993) have reviewed the data on the well-studied *Coma* cluster, for which they find the nucleonic mass fraction

$$f_{\rm N} \equiv \frac{M_{\rm N}}{M_{\rm tot}} \geq 0.009 + 0.05\,h^{-3/2}\ , \tag{5.65}$$

where the first term corresponds to the luminous matter in the cluster galaxies (within the Abell radius, $r_{\rm A} \approx 1.5\,h^{-1}$ Mpc) and the second to the intracluster X-ray emitting gas. Using results from hydrodynamical simulations of cluster formation these authors show that cooling and other dissipative effects could have enhanced $f_{\rm N}$ within $r_{\rm A}$ by a factor of at most $\Upsilon \approx 1.4$ over the global average. Similarly large nucleonic fractions (between 10% and 22%) have also been found in a sample of 13 other clusters (White and Fabian 1995). If such structures are indeed fair tracers of the universal mass distribution, then $f_{\rm N}$ is related to the global density parameters as

$$f_{\rm N} = \Upsilon \frac{\Omega_{\rm N}}{\Omega} \ . \tag{5.66}$$

Thus for $\Omega = 1$ as expected from inflation, the *Coma* observations can be consistent with standard BBN only for a low deuterium abundance (and low values of h) as shown in figure 18. Observations of large-scale structure and CMB anisotropy do favour high $\Omega_{\rm N}$ and low h for a critical density universe dominated by cold dark matter (e.g. White *et al* 1996, Adams *et al* 1996). Conversely if the deuterium abundance is indeed high, then to achieve consistency would require $\Omega \approx 0.1$ (for which there is, admittedly, independent observational evidence; see e.g. Coles and Ellis 1994). The dark matter in *Coma* and other clusters would then be comparable to that in the individual galactic halos. It is presently controversial whether this is indeed evidence for $\Omega < 1$ or whether the nucleonic enhancement factor Υ and/or the total cluster mass have been underestimated; also there may be sources of non-thermal pressure in clusters (viz. magnetic fields, cosmic rays) which would lower the inferred pressure of the X-ray emitting plasma, hence the value of $f_{\rm N}$ (see Felten and Steigman 1995).

In principle there may exist baryonic matter which does not participate in nuclear reactions and is utherefore unconstrained by the above arguments. Two examples are planetary mass black holes (Crawford and Schramm 1982, Hall and Hsu 1990) and strange quark nuggets (Witten 1984, see Alcock and Olinto 1988) which, it has been speculated, can be formed in cosmologically interesting amounts during a strongly first-order quark-hadron phase transition. As discussed earlier (§ 3.3.2), the fluctuations induced by such a violent phase transition should have resulted in the synthesis of observable (although rather uncertain) amounts of heavy elements (see Malaney and Mathews 1993). Also, according to our present theoretical understanding, this phase transition is relatively smooth (see Bonometto and Pantano 1993, Smilga 1995).

5.6.2. 'Non-Baryonic' dark matter: Given that the dark matter, at least in galactic halos, is unlikely to be baryonic, it is interesting to consider whether it may be composed of relic particles. This is well motivated (see Hall 1988, Sarkar 1991, Ellis 1994) since extensions of the Standard Model often contain new massive particles which are stable due to some new conserved quantum number. Alternatively, known particles which are cosmologically abundant, i.e. neutrinos, can constitute the dark matter if they acquire a small mass, e.g. through violation of global lepton number. Thus there are many particle candidates for the dark matter corresponding to many possible extensions of the SM (see Srednicki 1990). In order to optimize experimental strategies for their detection (see Primack *et al* 1988, Smith and Lewin 1990), it is important to narrow the field and it is here that constraints from cosmological nucleosynthesis play an important role.

It is generally assumed that dark matter particles must be weakly interacting since dark matter halos appear to be non-dissipative. However since there is as yet no 'standard' model of galaxy formation (see White 1994), it is legitimate to ask whether dark matter particles may have electromagnetic or strong interactions, given that their interaction lifetime exceeds the age of the Galaxy due to the low density of interstellar space (Goldberg and Hall 1986). This possibility has been studied in detail and various constraints identified (De Rújula, Glashow and Sarid 1990, Dimopoulos *et al* 1990, Chivukula *et al* 1990). According to the standard relic abundance calculation, such particles would have survived freeze-out with a *minimum* relic abundance of $\sim 10^{-12} - 10^{-10}$ per nucleon (Dover *et al* 1979, Wolfram 1979). These would have then bound with ordinary nuclei during nucleosynthesis, creating anomalously heavy isotopes of the light elements (Dicus and Teplitz 1980, Cahn and Glashow 1981). Sensitive searches for such isotopes have been carried out in a variety of terrestrial sites, all with negative results (see Rich *et al* 1987, Smith 1988). The best limits on the concentration of such particles are $\lesssim 10^{-29} - 10^{-28}$ per nucleon in the mass range $\sim 10 - 10^3\,\text{GeV}$ (Smith *et al* 1982), $\lesssim 10^{-24} - 10^{-20}$ per nucleon in the mass range $\sim 10^2 - 10^4\,\text{GeV}$ (Hemmick *et al* 1990) and $\lesssim 6 \times 10^{-15}$ in the mass range $\sim 10^4 - 10^8\,\text{GeV}$ (Verkerk *et al* 1992). Thus it is reasonable to infer that dark matter particles are electrically neutral and weakly interacting.†

Apart from the above general constraint, BBN would not appear to be relevant to individual particle candidates for the dark matter, since by definition their energy density is negligible relative to that of radiation during nucleosynthesis and furthermore, they are required to be stable or at least very long-lived. Nevertheless, BBN does provide another important constraint since it implies that the comoving entropy cannot have changed significantly (barring very exotic possibilities) since the MeV era. Thus the relic abundance of, for example, a 'cold dark matter' particle is unlikely to have been much altered from its value (4.16) at freeze-out, which we can rewrite as

$$\Omega_x h^2 \simeq \left(\frac{\langle\sigma v\rangle}{3 \times 10^{-10}\text{GeV}^{-2}}\right)^{-1} . \tag{5.67}$$

Thus it would be natural for a massive particle to constitute the dark matter if it is weakly interacting, i.e. if its interactions are fixed by physics above the Fermi scale. This is arguably the most direct hint we have today for an intimate connection between particle physics and cosmology, beyond their respective standard models.

† In principle, dark matter particles may be strongly *self*-interacting (Carlson *et al* 1992); this possibility is mildly constrained by the bound (4.15) on the speed-up rate. Nucleons themselves are strongly interacting so would be expected to have a negigibly small relic abundance from a state of thermal equilibrium. Their observed abundance then requires a primordial matter-antimatter *asymmetry*, which is $\sim 10^9$ times greater than their freeze-out abundance.

6. Conclusions

In the words of Ya'B Zeldovich, cosmology has long provided the "poor man's accelerator" for particle physics. As terrestrial accelerators come closer to the ultimate limits of technology and resources, it is imperative that our understanding of the cosmological laboratory be developed further, in particular since it offers probes of phenomena which can *never* be recreated in laboratories on Earth, however powerful our machines become. (This is not just to do with the energies available but because the early universe provides an *equilibrium* thermal environment, in contrast to the non-equilibrium environment of particle collisions in an accelerator.) There is an understandable reluctance, at least among experimentalists, to treat cosmological constraints on the same footing as as the results of repeatable and controlled laboratory experiments. However many theorists are already guided almost exclusively by cosmological considerations since there is simply no other experimental data available at the energies they are interested in. We therefore close with the following plea concerning the improvement of constraints from Big Bang nucleosynthesis.

In the comparison of the abundance data with the theoretical expectations, we have noted the rather unsatisfactory state of the observational situation today. Whereas there has been some concerted effort in recent years towards precise abundance determinations, the quoted numbers are still plagued by uncertain systematic errors and workers in this field use rather subjective criteria, e.g. "reasonable" and "sensible", to determine abundance bounds. In this regard, a comparison with the experimental style in high energy physics is illuminating. Thousands of person-years of effort have been invested in obtaining the precise parameters of the Z^0 resonance in e^+e^- collisions, which measures the number of light neutrino species (and other particles) which couple to the Z^0. In comparison, relatively little work has been done by a few small teams on measuring the primordial light element abundances, which provide a complementary check of this number as well as a probe of new superweakly interacting particles which do not couple to the Z^0. In our view, such measurements ought to constitute a *key* programme for cosmology, with the same priority as, say, the measurement of the Hubble constant or of the cosmological density parameter. Our understanding of the $2.73\,\mathrm{K}$ cosmic microwave background has been revolutionized by the accurate and consistent database provided by the *COBE* mission. A similar revolution is overdue for primordial nucleosynthesis.

Acknowledgments

I am indebted to many colleagues with whom I have enjoyed discussions and collaborations on cosmological constraints, in particular Steven Abel, Jeremy Bernstein, Ramanath Cowsik, John Ellis, Kari Enqvist, Graciela Gelmini, Peter Kernan, Roger Phillips, Graham Ross, Dennis Sciama and David Seckel. I am grateful to Bernard Pagel and Julie Thorburn for their critical remarks concerning elemental abundances, to Larry Kawano for the upgraded Wagoner computer code, and to all the authors who provided figures from their publicataions. I thank Michael Birkel, Herbi Dreiner and Cedric Lacey for carefully reading parts of the manuscript and all the people who provided valuable comments on the preprint version. Finally I would like to thank the editor, Richard Palmer, for his courtesey and patience.

This work was supported by an Advanced Fellowship awarded by the Particle Physics & Astronomy Research Council, UK.

References

Abel S A and Sarkar S 1995 *Phys. Lett.* B**342** 40
Abel S A, Sarkar S and Whittingham I B 1993 *Nucl. Phys.* B**392** 83
Abel S A, Sarkar S and White P L 1995 *Nucl. Phys.* B**454** 663
Adams J A, Ross G G and Sarkar S 1996 *Preprint* OUTP-96-48P (hep-ph/9608336) [submitted to *Phys. Lett.* B]
Adams, T F 1976 *Astron. Astrophys.* **50** 461
Adriani O *et al* (L3 collab.) 1993 *Phys. Rep.* **236** 1
Affleck I and Dine M 1985 *Nucl. Phys.* B**249** 361
Affleck I, Dine M and Seiberg N 1985 *Nucl. Phys.* B**256** 557
Aharonian F A, Kirillov-Ugryumov V G and Vardanian V V 1985 *Astrophys. Space Sci.* **115** 201
Ahlen S P *et al* 1987 *Phys. Lett.* B**195** 603
Akerlof C W and Srednicki M (ed) 1993 *Relativistic Astrophysics and Particle Cosmology: Proc. TEXAS/PASCOS Symp.* (New York: New York Academy of Sciences)
Albrecht A and Stebbins A 1992 *Phys. Rev. Lett.* **68** 2121
Alcock C and Olinto A 1988 *Annu. Rev. Nucl. Part. Sci.* **38** 161
Alcock C *et al* (MACHO collab.) 1995 *Phys. Rev. Lett.* **74** 2867
——1996 *Preprint* astro-ph/9606165 [submitted to *Astrophys. J.*]
Allahverdi R, Campbell B A and Olive K A 1994 *Phys. Lett.* B**341** 166
Allen C W 1973 *Astrophysical Quantities* (London: Athalone Press)
Alpher R A and Herman R C 1950 *Rev. Mod. Phys.* **22** 153
——1990 *Modern Cosmology in Retrospect* ed R Bertotti *et al* (Cambridge: Cambridge University Press) p 129
Alpher R A, Follin J W and Herman R C 1953 *Phys. Rev.* **92** 1347
Alvarez E, Dominguez-Tenreiro R D, Ibáñez Cabanell J M and Quirós M (ed) 1987 *Cosmology and Particle Physics: Proc. 17th GIFT Seminar on Theoretical Physics* (Singapore: World Scientific)
Alvarez-Gaume L, Polchinski J and Wise M 1983 *Nucl. Phys.* B**221** 495

Amaldi U, de Boer W and Furstenau H 1991 *Phys. Lett.* B**260** 447

Amati D, Konishi K, Meurice Y, Rossi G C and Veneziano G 1988 *Phys. Rep.* **162** 169

Antoniadis I 1991 *Particles, Strings and Cosmology: PASCOS-91* (Singapore: World Scientific) p 718

Antoniadis A, Bachas C, Lewellen D and Tomaras T 1988 *Phys. Lett.* B**207** 441

Antoniadis A, Benakli K and Quiròs M 1994 *Phys. Lett.* B**331** 313

Applegate J H, Hogan C J and Scherrer R J 1988 *Astrophys. J.* **329** 572

Armbruster B *et al* (KARMEN collab.) 1995 *Phys. Lett.* B**348** 19

Ashman K M 1992 *Pub. Astron. Soc. Pacific* **104** 1109

Astbury A, Campbell B A, Israel W, Khanna F C, Pinfold J L and Page D (ed) 1995 *Particle Physics and Cosmology: Proc. 9th Lake Louise Winter Institute* (Singapore: World Scientific)

Audouze J, Lindley D and Silk J 1985 *Astrophys. J.* **293** L53

Audouze J and Silk J 1989 *Astrophys. J.* **342** L5

Audouze J and Tinsley B M 1976 *Annu. Rev. Astron. Astrophys.* **14** 43

Aulakh C S and Mohapatra R N 1982 *Phys. Lett.* **119B** 136

Azeluos G *et al* 1986 *Phys. Rev. Lett.* **56** 2241

Baade D, Cristiani S, Lanz T, Malaney R A, Sahu K C and Vladilo G 1991 *Astron. Astrophys.* **251** 253

Babu K S and Rothstein I Z 1992 *Phys. Lett.* B**275** 112

Babu K S, Mohapatra R N and Rothstein I Z 1991 *Phys. Rev. Lett.* **67** 545

Baer H *et al* 1995 *Preprint* LBL-37016 (hep-ph/9503479)

Bagger J and Poppitz E 1993 *Phys. Rev.* **71** 2380

Bagger J, Poppitz E and Randall L 1994 *Nucl. Phys.* B**426** 3

Bahcall J N 1989 *Neutrino Astrophysics* (Cambridge: Cambridge University Press)

Bahcall J N and Ulrich R K 1988 *Rev. Mod. Phys.* **60** 297

Baier R, Pilon E, Pire B and Schiff D 1990 *Nucl. Phys.* B**336** 157

Bailin D and Love A 1986 *Introduction to Gauge Field Theory* (Bristol: Adam Hilger)

Balser D S, Bania T M, Brockway C J, Rood R T and Wilson T L 1994 *Astrophys. J.* **430** 667

Bamert P, Burgess C P and Mohapatra R N 1995 *Nucl. Phys.* B**438** 3

Banerjee B and Gavai R V 1992 *Phys. Lett.* B**293** 157

Bania T M, Rood R T and Wilson T L 1987 *Astrophys. J.* **323** 30

Banks T, Kaplan D and Nelson A E 1994 *Phys. Rev.* D**49** 779

Banks T, Berkooz M and Steinhardt P J 1995a *Phys. Rev.* D**52** 705

Banks T, Berkooz M, Shenker S H, Moore G and Steinhardt P J 1995b *Phys. Rev.* D**52** 3548

Barbieri R and Dolgov A D 1990 *Phys. Lett.* B**237** 440

——1991 *Nucl. Phys.* B**349** 742

Barbieri R, Ferrara S and Savoy C A 1982 *Phys. Lett.* **119B** 343

Barger V, Phillips R J N and Sarkar S 1995 *Phys. Lett.* B**352** 365 (erratum B**356** 617)

Barr S M, Chivukula R S and Farhi E 1990 *Phys. Lett.* B**241** 387

Barrow J D 1983 *Phys. Lett.* **125B** 377

——1987 *Phys. Rev.* D**35** 1805

Bartlett J G and Hall L J 1991 *Phys. Rev. Lett.* **66** 541

Beaudet G and Reeves H 1983 *Astron. Astrophys.* **134** 240

Beier E W *et al* 1992 *Phys. Lett.* B**283** 446

Bennett D P 1986 *Phys. Rev.* D**34** 3492

Bennett D P, Stebbins and A Bouchet F R 1992 *Astrophys. J.* **399** L5
Berezhiani Z G and Mohapatra R N 1993 *Phys. Rev.* D**52** 6607
Bergsma F *et al* (CHARM collab.) 1983 *Phys. Lett.* **128B** 361
Bernardi G *et al* (PS191 collab.) 1986 *Phys. Lett.* **166B** 479
Bernstein J 1988 *Kinetic Theory in the Expanding Universe* (Cambridge: Cambridge University Press)
Bernstein J, Brown L M and Feinberg G 1989 *Rev. Mod. Phys.* **61** 25
Bertolami O 1988 *Phys. Lett.* B**209** 277
Bertolini S and Steigman G 1992 *Nucl. Phys.* B**387** 193
Bethke S and Pilcher J E 1992 *Annu. Rev. Nucl. Part. Sci.* **42** 251
Bilenky S and Petcov S T 1987 *Rev. Mod. Phys.* **59** 671
Binétruy P 1985 *Proc. 6th Workshop on Grand Unification* ed S Rudaz and T Walsh (Singapore: World Scientific) p 403
Binétruy P and Dudas E 1995 *Nucl. Phys.* B**442** 21
Binney J and Tremaine S 1987 *Galactic Dynamics* (Princeton: Princeton University Press)
Birkinshaw M and Hughes J P 1994 *Astrophys. J.* **420** 33
Black D C 1971 *Nature* **231** 1480
Blandford R D and Narayan R 1992 *Annu. Rev. Astron. Astrophys.* **30** 311
Blumenthal G R and Gould R J 1970 *Rev. Mod. Phys.* **42** 237
Boehm F *et al* 1991 *Phys. Lett.* B**255** 143
Boesgaard A M and Steigman G 1985 *Annu. Rev. Astron. Astrophys.* **23** 919
Bonometto S A and Pantano O 1993 *Phys. Rep.* **228** 175
Bopp P *et al* 1986 *Phys. Rev. Lett.* **56** 919
Börner G 1988 *The Early Universe* (Berlin: Springer-Verlag)
Bouquet A and Salati P 1987 *Nucl. Phys.* B**284** 557
Bowler M and Jelley N A 1994 *Phys. Lett.* B**331** 193
Boyanovsky D, de Vega H J, Holman R and Salgado J F J 1996 *Preprint* astro-ph/9609007
Boyd G *et al* 1995 *Phys. Rev. Lett.* **75** 4169
Brandenberger R 1985 *Rev. Mod. Phys.* **57** 1
——1991 *Phys. Scr.* T**36** 114
——1994 *Int. J. Mod. Phys.* A**9** 2117
Brandenberger R, Albrecht A and Turok N 1986 *Nucl. Phys.* B**277** 655
Brignole A, Espinosa J R, Quirós M and Zwirner F 1994 *Phys. Lett.* B**324** 181
Britton D I *et al* 1992 *Phys. Rev.* D**46** 885
——1994 *Phys. Rev.* D**49** 28
Brocklehurst M 1972 *Mon. Not. Roy. Astron. Soc.* **157** 211
Brown L 1992 *Astrophys. J.* **389** 251
Brustein R and Steinhardt P J 1993 *Phys. Lett.* B**302** 196
Bryman D A 1993 *Comm. Nucl. Part. Phys.* **21** 101
Bryman D A *et al* 1983 *Phys. Rev. Lett.* **50** 1546
Buchmüller W and Philipsen O 1995 *Nucl. Phys.* B**443** 47
Burkhardt H and Steinberger J 1991 *Annu. Rev. Nucl. Part. Sci.* **41** 55
Burles S and Tytler D 1996 *Preprint* astro-ph/9603070 [submitted to *Science*]
Busculic D *et al* (ALEPH collab.) 1995 *Phys. Lett.* B**349** 585
Byrne J 1982 *Rep. Prog. Phys.* **45** 115

Cahn R and Glashow S 1981 *Science* **213** 607

Caldwell D O *et al* 1988 *Phys. Rev. Lett.* **61** 510

Caldwell R R and Allen B 1992 *Phys. Rev.* D**45** 3447

Cambier J-L, Primack J and Sher M 1982 *Nucl. Phys.* B**209** 372

Campbell B A, Davidson S, Ellis J and Olive K A 1991 *Phys. Lett.* **B256** 457

——1992a *Phys. Lett.* **B297** 118

——1992b *Astropart. Phys.* **1** 77

Cardall C Y and Fuller G M 1996a *Preprint* astro-ph/9603071

——1996b *Phys. Rev.* D**54** 1260

Carlson E D and Glashow S L 1987 *Phys. Lett.* B**193** 168

Carlson E D, Esmailzadeh R, Hall L J and Hsu S D H 1990 *Phys. Rev. Lett.* **65** 2225

Carlson E D, Machacek M and Hall L J 1992 *Astrophys. J.* **398** 43

Carr B J 1994 *Annu. Rev. Astron. Astrophys.* **32** 531

Carroll S M, Press W H and Turner E L 1992 *Annu. Rev. Astron. Astrophys.* **30** 499

Carswell R F, Rauch M, Weymann R J, Cooke A J and Webb J K 1994 *Mon. Not. Roy. Astron. Soc.* **268** L1

Carswell R F *et al* 1996 *Mon. Not. Roy. Astron. Soc.* **278** 506

Caughlan G R and Fowler W A 1988 *Atom. Data Nucl. Data Tables* **40** 283

Caughlan G R, Fowler W A, Harris M J and Zimmerman B A 1985 *Atom. Data Nucl. Data Tables* **32** 197

Cen R, Ostriker J P and Peebles J 1993 *Astrophys. J.* **415** 423

Chaboyer B and Demarque P 1994 *Astrophys. J.* **433** 510

Chamseddine A, Arnowitt R and Nath P 1982 *Phys. Rev. Lett.* **49** 970

Chang S and Choi K 1993 *Phys. Lett.* B**316** 51

——1994 *Phys. Rev.* D**49** 12

Charbonnel C 1995 *Astrophys. J.* **453** L41

Cheng H-Y 1988 *Phys. Rep.* **158** 1

Cheng T-P and Li L-F 1984 *Gauge Theory of Elementary Particle Physics* (Oxford: Oxford University Press)

Chikashige Y, Mohapatra R N and Peccei R 1980 *Phys. Rev. Lett.* **45** 1926

Chivukula R S and Walker T P 1990 *Nucl. Phys.* B**329** 445

Chivukula R S, Cohen A G, Dimopoulos S and Walker T P 1990 *Phys. Rev. Lett.* **65** 957

Chivukula R S, Cohen A G, Luke M and Savage M J 1993 *Phys. Lett.* B**298** 380

Chivukula R S, Dugan M J, Golden M and Simmons E H 1995 *Annu. Rev. Nucl. Part. Sci.* **45** 255

Choi K and Kim J E 1985 *Phys. Lett.* **165B** 71

Clayton D D 1985 *Astrophys. J.* **290** 428

Clegg R E S 1983 *Mon. Not. Roy. Astron. Soc.* **229** 31P

Cline J M 1992 *Phys. Rev. Lett.* **68** 3137

Cline J M and Walker T P 1992 *Phys. Rev. Lett.* **68** 270

Cline J M, Kainulainen K and Olive K A 1994 *Phys. Rev.* **D49** 6394

Cohen A G, Kaplan D B and Nelson A E 1994 *Annu. Rev. Nucl. Part. Sci.* **47** 27

Coleman S and Weinberg E 1973 *Phys. Rev.* D**7** 1888

Coles P and Ellis G F R 1994 *Nature* **370** 609

Collins P D B, Martin A D and Squires E J 1989 *Particle Physics and Cosmology* (New York: Wiley)

Cooper-Sarkar A M *et al* (BEBC WA66 collab.) 1985 *Phys. Lett.* **160B** 207
Cooper-Sarkar A M, Sarkar S, Guy J, Venus W, Hulth P O and Hultqvist K 1992 *Phys. Lett.* B**280** 153
Copi C J, Schramm D N and Turner M S 1995a *Science* **267** 192
——1995b *Phys. Rev. Lett.* **75** 3981
——1995c *Astrophys. J.* **455** L95
——1996 *Preprint* FERMILAB-Pub-96/122-A [astro-ph/9606059 [submitted to *Phys. Rev. Lett.*]
Costa G and Zwirner F 1986 *Riv. Nuovo Cim.* **9** 1
Coughlan G, Fischler W, Kolb E W, Raby S and Ross G G 1983 *Phys. Lett.* **131B** 59
Coughlan G, Holman R, Ramond P and Ross G G *Phys. Lett.* **140B** 44
Coughlan G, Ross G G, Holman R, Ramond P, Ruiz-Altaba M and Valle J W F 1985 *Phys. Lett.* **158B** 401
Coulson D, Ferreira P, Graham P and Turok N 1994 *Nature* **368** 27
Cowsik R 1981 *Cosmology and Particles* ed J Audouze *et al* (Gif Sur Yvette: Editions Frontiéres) p 157
Cowsik R and McCleland J 1972 *Phys. Rev. Lett.* **29** 669
Crawford M and Schramm D N 1982 *Nature* **298** 538
Cremmer E, Julia B, Scherk J, Ferrara S, Girardello L and van Nieuwenhuizen P 1979 *Nucl. Phys.* B**147** 105
Cremmer E, Ferrara S, Girardello L and van Proyen A 1983 *Nucl. Phys.* B**212** 413
Dar A 1995 *Astrophys. J.* **449** 550
David Y and Reeves H 1980 *Phil. Trans. R. Soc.* A**296** 415
Davidson K and Kinman T D 1985 *Astrophys. J. Suppl.* **58** 321
Davidson S and Peskin M 1994 *Phys. Rev.* D**49** 2114
Dearborn D S P, Schramm D N and Steigman G 1986 *Astrophys. J.* **302** 35
Dearborn D S P, Schramm D N, Steigman G and Truran J 1989 *Astrophys. J.* **347** 455
de Boer W 1994 *Prog. Part. Nucl. Phys.* **33** 201
de Carlos B, Casas J A, Quevedo F and Roulet E 1993 *Phys. Lett.* B**318** 447
Dekel A 1994 *Annu. Rev. Astron. Astrophys.* **32** 371
de Laix A A and Scherrer R J 1993 *Phys. Rev.* D**48** 562
Delbourgo-Salvador P, Gry C, Malinie G and Audouze J, 1985 *Astron. Astrophys.* **150** 53
Delbourgo-Salvador P, Audouze J and Vidal-Madjar A 1987 *Astron. Astrophys.* **174** 365
De Leener-Rosier N *et al* 1991 *Phys. Rev.* D**43** 3611
Deliyannis C P, Demarque P, Kawaler S D, Krauss, L M and Romanelli P 1989 *Phys. Rev. Lett.* **62** 1583
Deliyannis C P, Demarque P and Kawaler S D 1990 *Astrophys. J. Suppl.* **73** 21
Deliyannis C P, Pinsonneault M H and Duncan D K 1993 *Astrophys. J.* **414** 740
Denegri D, Sadoulet B and Spiro M 1990 *Rev. Mod. Phys.* **62** 1
Derbin A V *et al* 1994 *Yad. Phys.* **57** 236
Derendinger J-P and Savoy C A 1984 *Nucl. Phys.* B**237** 307
Derendinger J-P, Ibáñez L and Nilles H P 1985 *Phys. Lett.* **155B** 65
De Rújula A and Glashow S 1981 *Phys. Rev. Lett.* **45** 942
De Rújula A, Nanopoulos D V and Shaver P A (ed) 1987 *An Unified View of the Macro- and Micro-Cosmos* (Singapore: World Scientific)
De Rújula A, Glashow S and Sarid U 1990 *Nucl. Phys.* B**333** 173

Deser S and Zumino B 1977 *Phys. Rev. Lett.* **38** 1433

Dicke R H and and Peebles P J E 1979 *General Relativity: An Einstein Centenary Survey* ed S W Hawking and W Israel (Cambridge: Cambridge University Press) p 504

Dicus D A and Teplitz V L 1980 *Phys. Rev. Lett.* **44** 218

Dicus D A, Kolb E W and Teplitz V L 1977 *Phys. Rev. Lett.* **39** 168

——1978a *Astrophys. J.* **221** 327

Dicus D A, Kolb E W, Teplitz V L and Wagoner R V 1978b *Phys. Rev.* D**17** 1529

Dicus D A, Kolb E W, Gleeson A M, Sudarshan E C G, Teplitz V L and Turner M S 1982 *Phys. Rev.* D**26** 2694

Dimopoulos S 1995 *Proc XXVII Conf. on High Energy Physics, Glasgow* ed P J Bussey and I G Knowles (Bristol: IOP Publishing) Vol I, p 93

Dimopoulos S and Hall L J 1987 *Phys. Lett.* **196B** 135

Dimopoulos S and Turner M S 1982 *The Birth of the Universe* ed J Audouze and T Tran Thanh Van (Gif Sur Yvette: Editions Frontiéres) p 113

Dimopoulos S, Esmailzedeh R, Hall L J and Starkman G D 1988 *Astrophys. J.* **330** 545

——1989 *Nucl. Phys.* B**311** 699

Dimopoulos S, Eichler D, Esmailzadeh R and Starkman G 1990 *Phys. Rev.* D**41** 2388

Dimopoulos S, Giudice G F and Tetradis N 1995 *Nucl. Phys.* B**454** 59

Dine M (ed) 1988 *String Theory in Four Dimensions* (Amsterdam: North-Holland)

——1990 *Annu. Rev. Nucl. Part. Sci.* **40** 145

Dine M, Fischler W and Nemechansky D 1984 *Phys. Lett.* **136B** 169

Dine M, Kaplunovsky V, Mangano M, Nappi C and Seiberg N 1985 *Nucl. Phys.* **B259** 519

Dine M and Nelson A E 1993 *Phys. Rev.* D**48** 1277

Dine M, Randall L and Thomas S 1995 *Phys. Rev. Lett.* **75** 398

——1996 *Nucl. Phys.* B**458** 291

Dine M, Nelson A E, Nir Y and Shirman Y 1996 *Phys. Rev.* D**53** 2658

Dixon L J and Nir, Y 1991 *Phys. Lett.* B**266** 425

Dodelson S 1989 *Phys. Rev.* D**40** 3252

Dodelson S and Turner M S 1992 *Phys. Rev.* D**46** 3372

Dodelson S, Guyk G and Turner M S 1994 *Phys. Rev.* D**49** 5068

Dolgov A D 1992 *Phys. Rep.* **222** 309

Dolgov A D and Fukugita M 1992 *Phys. Rev.* D**46** 5378

Dolgov A D and Rothstein I Z 1993 *Phys. Rev. Lett.* **71** 476

Dolgov A D and Zel'dovich Ya B 1981 *Rev. Mod. Phys.* **53** 1

Dolgov A D, Kainulainen K and Rothstein I Z 1995 *Phys. Rev.* D**51** 4129

Dolgov A D, Pastor S and Valle J W F 1996 *Phys. Lett.* B**383** 193

Dominguez-Tenreiro R 1987 *Astrophys. J.* **313** 523

Dover C B, Gaisser T K and Steigman G 1979 *Phys. Rev. Lett.* **42** 1117

Drees M and Nojiri M M 1993 *Phys. Rev.* D**47** 376

Dreiner H and Ross G G 1993 *Nucl. Phys.* B**410** 188

Dubbers D 1991 *Prog. Part. Nucl. Phys.* **26** 173

Duncan D K, Lambert D L and Lemke M 1992 *Astrophys. J.* **401** 584

Dvali G 1995 *Preprint* IFUP-TH 09/95 (hep-ph/9503259) ?

Edmunds M G 1994 *Mon. Not. Roy. Astron. Soc.* **270** L37

Efstathiou G P 1990 *Physics of the Early Universe* ed J Peacock *et al* (Bristol: Adam Hilger) p 361
Elliott T, King S F and White P L 1994 *Phys. Rev.* D**49** 2435
Ellis J 1987 *Phys. Scr.* T**15** 61
——1993 *Preprint* CERN-TH-7083-93 [to appear in *Les Houches Session 60: Cosmology and Large Scale Structure*]
——1994 *Nuovo Cimento* **107A** 1091
——1995 *International Symposium on Lepton Photon Interactions, Beijing* ed Z-P Zheng and H-S Chen (Singapore: World Scientific)
Ellis J and Olive K A 1983 *Nucl. Phys.* B**223** 252
Ellis J and Steigman G 1979 *Phys. Lett.* **89B** 186
Ellis J, Nanopoulos D V and Rudaz S 1982 *Nucl. Phys.* B**202** 43
Ellis J, Linde A D and Nanopoulos D V 1983 *Phys. Lett.* **118B** 59
Ellis J, Hagelin J S, Nanopoulos D V, Olive K A and Srednicki M 1984a *Nucl. Phys.* B**238** 453
Ellis J, Kim J E and Nanopoulos D V 1984b *Phys. Lett.* **145B** 181
Ellis J, Gelmini G B, Jarlskog C, Ross G G and Valle J W F 1985a *Phys. Lett.* **150B** 142
Ellis J, Nanopoulos D V and Sarkar S 1985b *Nucl. Phys.* B**259** 175
Ellis J, Enqvist K, Nanopoulos D V, Olive K A, Quirós M and Zwirner F 1986a *Phys. Lett.* B**176** 403
Ellis J, Enqvist K, Nanopoulos D V and Sarkar S 1986b *Phys. Lett.* **167B** 457
Ellis J, Nanopoulos D V and Quirós M 1986c *Phys. Lett.* B**174** 176
Ellis J, Enqvist K, Nanopoulos D V and Olive K A 1987 *Phys. Lett.* B**191** 343
Ellis J, Gunion J, Haber H, Roszkowski L and Zwirner F 1989 *Phys. Rev.* D**39** 844
Ellis J, Kelly S and Nanopoulos D V 1991 *Phys. Lett.* B**249** 441
Ellis J, Gelmini G B, Nanopoulos D V, Lopez J and Sarkar S 1992 *Nucl. Phys.* B**373** 399
Ellis J, Nanopoulos D V, Olive K A and Rey S-J 1996 *Astropart. Phys.* **4** 371
Ellwanger U 1983 *Phys. Lett.* **133B** 187
Elmfors P, Enqvist K and Vilja I 1994 *Phys. Lett.* B**326** 37
Enqvist K 1986 *Proc. 2nd ESO-CERN Symp. on Cosmology, Astronomy and Fundamental Physics* ed G Setti and L van Hove (Garching: ESO) p 137
Enqvist K and Eskola K J 1990 *Mod. Phys. Lett.* A**5** 1919
Enqvist K and Nanopoulos D V 1986 *Prog. Part. Nucl. Phys.* **16** 1
Enqvist K and Sirkka J 1993 *Phys. Lett.* B**314** 298
Enqvist K and Uibo H 1993 *Phys. Lett.* B**301** 376
Enqvist K, Kainulainen K and Maalampi J 1990a *Phys. Lett.* B**244** 186
——1990b *Phys. Lett.* B**249** 531
——1991 *Nucl. Phys.* B**349** 754
Enqvist K, Kainulainen K and Semikoz V 1992a *Nucl. Phys.* B**374** 392
Enqvist K, Kainulainen K and Thomson M 1992b *Nucl. Phys.* B**373** 498
——1992c *Phys. Rev. Lett.* **68** 744
——1992d *Phys. Lett.* B**280** 245
Epstein R, Lattimer J and Schramm D N 1976 *Nature* **263** 198
Escribano R, Massó E and Toldrà R 1995 *Phys. Lett.* B**356** 313
Esmailzadeh R, Starkman G D and Dimopoulos S 1991 *Astrophys. J.* **378** 504
Faber S and Gallagher J S 1979 *Annu. Rev. Astron. Astrophys.* **17** 135
Faraggi A and Nanopoulos D V 1991 *Mod. Phys. Lett.* A**6** 61

Fargion D and Roos M 1984 *Phys. Lett.* **147B** 34

Fargion D and Shepkin M G 1981 *Phys. Lett.* **146B** 46

Farhi E and Susskind L 1981 *Phys. Rep.* **74** 277

Farrar G and Fayet P 1978 *Phys. Lett.* **76B** 575

Faul D D, Berman B L, Meyer P and Olson D L 1981 *Phys. Rev.* **24** 84

Fayet P 1979 *Phys. Lett.* **86B** 272

——1982 *Electroweak Interactions and Grand Unified Theories* ed J Tran Thanh Van (Gif Sur Yvette: Editions Frontiéres) p 161

——1984 *Phys. Rep.* **105** 21

——1987 *Phys. Scr.* T**15** 46

Fayet P and Ferrara S 1975 *Phys. Rep.* **32** 249

Felten J E and Steigman G 1995 *Space Sci. Rev.* **74** 245

Ferrara S, Kounnas C, Porrati M and Zwirner F 1989 *Nucl. Phys.* B**318** 75

Fields B D and Olive K A 1996 *Phys. Lett.* B**368** 103

Fields B D, Dodelson S and Turner M S 1993 *Phys. Rev.* D**47** 4309

Fields B D, Kainulainen K and Olive K A 1995 *Preprint* CERN-TH/95-335 (hep-ph/9512321) [*Astropart. Phys.*, in press]

Fields B D, Kainulainen K, Olive K A and Thomas D 1996 *Preprint* CERN-TH/96-59 (astro-ph/9603009)

Fischler W 1994 *Phys. Lett.* B**332** 277

Fischler W, Giudice G F, Leigh R G and Paban S 1991 *Phys. Lett.* B**258** 45

Foot R and Volkas R R 1995a *Phys. Rev. Lett.* **75** 4350

——1995b *Phys. Rev.* D**52** 6595

Foot R, Thomson M and Volkas R R 1996 *Phys. Rev.* D**53** 5349

Fowler W A and Hoyle F 1964 *Nature* **203** 1108

Fowler W A, Caughlan G R and Zimmerman B A 1975 *Annu. Rev. Astron. Astrophys.* **13** 69

Freedman S 1990 *Comm. Nucl. Part. Phys.* **19** 209

Freese K, Kolb E W and Turner M S 1983 *Phys. Rev.* D**27** 1689

Frieman J A and Giudice G 1989 *Phys. Lett.* B**224** 125

Frieman J A, Kolb E W and Turner M S 1990 *Phys. Rev.* D**41** 3080

Fukugita M, Nötzold D, Raffelt G and Silk J 1988 *Phys. Rev. Lett.* **60** 879

Fukugita M, Hogan C J and Peebles P J E 1993 *Nature* **366** 309

Fukugita M and Yanagida T 1986 *Phys. Lett.* **174B** 45

——1994 *Physics and Astrophysics of Neutrinos* ed A Suzuki (Tokyo: Springer Verlag) p 248

Fukugita M and Yazaki 1987 *Phys. Rev.* D**36** 3817

Fuller G M, Boyd R N and Kalen J D 1991 *Astrophys. J.* **371** L11

Fuller G M and Malaney R A 1991 *Phys. Rev.* D**43** 3136

Galli D, Palla F, Ferrini F and Penco U 1995 *Astrophys. J.* **443** 536

Gari M and Hebach H 1981 *Phys. Rep.* **72** 1

Geiss J 1993 *Origin and Evolution of the Elements* ed N Prantzos *et al* (Cambridge: Cambridge University Press) p 89

Geiss J and Reeves H 1972 *Astron. Astrophys.* **18** 126

Gelmini G, Nussinov S and Peccei R 1992 *Int. J. Mod. Phys.* A**7** 3141

Gelmini G and Roncadelli M 1981 *Phys. Lett.* **99B** 411

Gelmini G and Roulet E 1995 *Rep. Prog. Phys.* **58** 1207

German G and Ross G G 1986 *Phys. Lett.* B**172** 305

Gershteĭn S and Zel'dovich Ya B 1966 *JETP Lett.* **4** 120

Gherghetta T 1996 *Preprint* UM-TH-96-10 (hep-ph/9607448)

Gibbons G, Hawking S W and Siklos S T C (ed) 1983 *The Very Early Universe* (Cambridge: Cambridge University Press)

Gilmore G, Gustafsson B, Edvardsson B and Nissen P E 1992 *Nature* **357** 379

Giudice G 1990 *Phys. Lett.* B**251** 460

Giudice G and Masiero A 1988 *Phys. Lett.* B**206** 480

Giveon A, Porrati M and Rabinovici E 1994 *Phys. Rep.* **244** 77

Gloeckler G and Geiss J 1996 *Nature* **381** 210

Gnedin N Y and Ostriker J P 1992 *Astrophys. J.* **400** 1

Goldberg H and Hall L J 1986 *Phys. Lett.* B**174** 151

Goncharov A S, Linde A D and Vysotskiĭ M I 1984 *Phys. Lett.* **147B** 279

Gondolo P and Gelmini G 1991 *Nucl. Phys.* B**360** 145

Gondolo P, Gelmini G and Sarkar S 1993 *Nucl. Phys.* B**392** 111

Gonzalez-Garcia M C and Valle J W F 1990 *Phys. Lett.* B**240** 163

Gott J R, Gunn J E, Schramm D N and Tinsley B M 1974 *Astrophys. J.* **194** 543

Govaerts J, Lucio J L, Martinez A and Pestiau J 1981 *Nucl. Phys.* A**368** 409

Grasso D and Kolb E W 1996 *Phys. Rev.* D**54** 1374

Gratsias J, Scherrer R J and Spergel D N 1991 *Phys. Lett.* B**262** 298

Green M, Schwarz J H and Witten E 1987 *Superstring Theory* (Cambridge: Cambridge University Press)

Grifols J A and Massó E 1987 *Mod. Phys. Lett.* A**2** 205

Gronau M and Yahalom R 1984 *Phys. Rev.* D**30** 2422

Gross D J, Pisarski R D and Yaffe, L G 1981 *Rev. Mod. Phys.* **53** 43

Gross D J, Harvey J A, Martinec E and Rohm R 1985 *Phys. Rev. Lett.* **54** 502

Guth A 1981 *Phys. Rev.* D**23** 347

Gyuk G and Turner M S 1993 *Preprint* FERMILAB-Pub-93/181-A (astro-ph/9307025)

——1994 *Phys. Rev.* D**50** 6130

Haber H and Kane G 1985 *Phys. Rep.* **117** 75

Haber H and Weldon A 1981 *Phys. Rev.* D**25** 502

Hagelin J S and Parker R J D 1990 *Nucl. Phys.* B**329** 464

Hall L J 1988 *Proc. 16th SLAC Summer Institute on Particle Physics* ed E C Brennan (Stanford: SLAC) p 85

Hall L and Hsu S 1990 *Phys. Rev. Lett.* **64** 2848

Hall L and Suzuki M 1984 *Nucl. Phys.* B**231** 419

Hall L, Lykken J and Weinberg S 1983 *Phys. Rev.* D**27** 2359

Halm I 1987 *Phys. Lett.* B**188** 403

Hannestad S and Madsen J 1995 *Phys. Rev.* D**52** 1764

——1996 *Phys. Rev. Lett.* **76** 2848; erratum hep-ph/9606452

Harris M J, Fowler W A, Caughlan G R and Zimmerman B A 1983 *Annu. Rev. Astron. Astrophys.* **21** 165

Harrison E R 1973 *Annu. Rev. Astron. Astrophys.* **11** 155

——1993 *Astrophys. J.* **403** 28

Harvey J A, Kolb E W, Reiss D B and Wolfram S 1981 *Nucl. Phys.* B**177** 456

Hata N, Scherrer R J, Steigman G, Thomas D, Walker T P, Bludman S and Langacker P 1995 *Phys. Rev. Lett.* **75** 3977

Hata N, Scherrer R J, Steigman G, Thomas D and Walker T P 1996a *Astrophys. J.* **458** 637

Hata N, Steigman G, Bludman S and Langacker P 1996b *Preprint* OSU-TA-6/96 (astro-ph/9603087)

Hawking S and Ellis G F R 1973 *The Large-scale Structure of Space-time* (Cambridge: Cambridge University Press)

Hayashi C 1950 *Prog. Theo. Phys.* **5** 224

Hecht H F 1971 *Astrophys. J.* **170** 401

Hemmick T K *et al* 1990 *Phys. Rev.* D**41** 2074

Herrera M A and Hacyan S 1989 *Astrophys. J.* **336** 539

Hewett, J L and Rizzo T G 1989 *Phys. Rep.* **183** 193

Hime A 1992 *Mod. Phys. Lett.* A**7** 1301

Hime A and Jelley N 1991 *Phys. Lett.* B**257** 441

Hime A, Phillips R J N, Ross G G and Sarkar S 1991 *Phys. Lett.* B**260** 381

Hinchliffe I (ed) 1987 *Cosmology and Particle Physics: Proc. Theoretical Workshop, Berkeley* (Singapore: World Scientific)

Hindmarsh M B and Kibble T W B 1995 *Rep. Prog. Phys.* **58** 477

Hobbs L and Pilachowski C 1988 *Astrophys. J.* **334** 734

Hobbs L and Thorburn J A 1991 *Astrophys. J.* **375** 116

——1994 *Astrophys. J.* **428** L25

Hodges H and Turner M S 1988 *Preprint* FERMILAB-PUB-88/115-A-Rev

Hogan C J 1994 *Advances in Astrofundamental Physics* ed N Sanchez and A Zichichi (Singapore: World Scientific) p 38

——1995a *Cosmic Abundances* ed S Holt and G. Sonneborn (San Francisco: PASP) p 67

——1995b *Astrophys. J.* **441** L17

——1996 *Phys. Rev.* D**54** 112

Hogan C J and Rees M 1984 *Nature* **311** 109

Holman R, Ramond P and Ross G G 1984 *Phys. Lett.* **137B** 343

Hosotani Y 1981 *Nucl. Phys.* B**191** 411

Hoyle F and Tayler R J 1964 *Nature* **203** 1108

Hu W and Silk J 1993 *Phys. Rev. Lett.* **70** 2661

Huchra J P 1992 *Science* **256** 321

Ibáñez L 1994 *Proc. Intern. EPS Conf. on High Energy Physics, Marseilles* ed J Carr and M Perrottet (Paris: Editions Frontiéres) p 911

Ibáñez L and Ross G G 1993 *Perspectives on Higgs Physics* ed G Kane (Singapore: World Scientific) p 229

Iben I and Truran J W 1978 *Astrophys. J.* **220** 980

Illarianov A F and Sunyaev R A 1975 *Sov. Astron.* **18** 413

Izotov Y I, Thuan T X and Lipovetsky V A 1994 *Astrophys. J.* **435** 647

——1996 *Preprint* (submitted to *Astrophys. J.)*

Jacoby G H, Branch D, Ciardullo R, Davies R, Harris W E, Pierce M J, Pritchet C J, Tonry J L and Welch D L 1992 *Pub. Astron. Soc. Pacific* **104** 599

Jakobsen P, Boksenberg A, Deharveng J M, Greenfield P, Jedrzejewski and Paresce F 1994 *Nature* **370** 35

Jedamzik K, Fuller G M and Mathews G J 1994a *Astrophys. J.* **423** 50

Jedamzik K, Fuller G M, Mathews G J and Kajino T 1994b *Astrophys. J.* **422** 423

Juszkiewicz R, Silk J and Stebbins A 1985 *Phys. Lett.* **158B** 463

Kainulainen K 1990 *Phys. Lett.* B**244** 191

Kajino T, Toki H and Austin S M 1987 *Astrophys. J.* **319** 531

Kajantie K, Laine M Rummukainen K and Shaposhnikov M 1996 *Nucl. Phys.* B**466** 189; *Preprint* CERN-TH-96-126 (hep-ph/9605288)

Kane G 1987 *Modern Elementary Particle Physics* (Redwood City, CA: Addison-Wesley)

Kang H-S and Steigman G 1992 *Nucl. Phys.* B**372** 494

Kapusta J 1988 *Finite Temperature Field Theory* (Cambridge: Cambridge University Press)

Kaul R K 1983 *Rev. Mod. Phys.* **55** 449

Kawano L 1988 *Preprint* FERMILAB-Pub-88/34-A

——1992 *Preprint* FERMILAB-Pub-92/04-A

Kawano L, Schramm D N and Steigman G 1988 *Astrophys. J.* **327** 750

Kawano L, Fuller G M, Malaney R A and Savage M J 1992 *Phys. Lett.* B**275** 487

Kawasaki M and Moroi T 1995a *Prog. Theo. Phys.* **93** 879

——1995b *Astrophys. J.* **452** 506

——1995c *Phys. Lett.* B**346** 27

Kawasaki M and Sato K 1987 *Phys. Lett.* B**189** 23

Kawasaki M, Terasawa N and Sato K 1986 *Phys. Lett.* B**178** 71

Kawasaki M, Kernan P, Kang H-S, Scherrer, R J, Steigman G and Walker T P 1994 *Nucl. Phys.* B**419** 105

Kayser B, Gibrat-Debu F and Perrier F 1989 *The Physics of Massive Neutrinos* (Singapore: World Scientific)

Kennicutt R C, Freedman W L and Mould J R 1995 *Astron. J.* **110** 1476

Kernan P J 1993 *PhD thesis* (Ohio State University)

Kernan P J and Krauss L M 1994 *Phys. Rev. Lett.* **72** 3309

Kernan P J and Sarkar S 1996 *Phys. Rev.* D**54** 3681

Khlopov M Yu and Linde A D 1984 *Phys. Lett.* **138B** 265

Khlopov M Yu and Petcov S 1981 *Phys. Lett.* **99B** 117 (erratum **100B** 520)

Kibble T W B 1976 *J. Phys. A: Math. Gen.* **9** 1387

Kim J E 1987 *Phys. Rep.* **150** 1

Kim J E and Nilles H P 1984 *Phys. Lett.* **138B** 150

King S F 1995 *Rep. Prog. Phys.* **58** 263

Kingdon J and Ferland G J 1995 *Astrophys. J.* **442** 714

Kofman L, Linde A D and Starobinsky A A 1996 *Phys. Rev. Lett.* **76** 1011

Kolb E W 1980 *Proc. Telemark Workshop on Neutrino Mass* ed V Barger and D Cline (Madison: University of Wisconsin) p 61

Kolb E W and Goldman T 1979 *Phys. Rev. Lett.* **43** 897

Kolb E W and Peccei R D (ed) 1995 *Particle and Nuclear Astrophysics and Cosmology in the Next Millenium: Snowmass '94* (Singapore: World Scientific)

Kolb E W and Scherrer R J 1982 *Phys. Rev.* D**25** 1481

Kolb E W and Turner M S 1983 *Annu. Rev. Nucl. Part. Sci.* **33** 645
——1990 *The Early Universe* (Redwood City, CA: Addison-Wesley)
Kolb E W, Seckel D and Turner M S 1985 *Nature* **314** 415
Kolb E W, Perry M J and Walker T P 1986a *Phys. Rev.* D**33** 869
Kolb E W, Turner M S, Lindley D, Olive K A and Seckel D (ed) 1986b *Inner Space–Outer Space* (Chicago: University of Chicago Press)
Kolb E W, Turner M S and Walker T P 1986c *Phys. Rev.* D**34** 2197
Kolb E W, Turner M S and Schramm D N 1989 *Neutrino Physics* ed K Winter (Cambridge: Cambridge University Press) p 239
Kolb E W, Turner M S, Chakravorty A and Schramm D N 1991 *Phys. Rev. Lett.* **67** 533
Kounnas C, Zwirner F and Pavel I 1994 *Phys. Lett.* B**335** 403
Krakauer D A *et al* 1990 *Phys. Lett.* B**252** 177
——1991 *Phys. Rev.* D**44** R6
Krauss L M 1983a *Nucl. Phys.* B**227** 1303
——1983b *Phys. Lett.* **128B** 37
——1984 *Phys. Rev. Lett.* **53** 1976
——1985 *Preprint* HUTP-85-A040
Krauss L M and Kernan P 1994 *Astrophys. J.* **432** L79
——1995 *Phys. Lett.* B**347** 347
Krauss L M and Rey S-J (ed) 1992 *Baryon Number Violation at the Electroweak Scale* (Singapore: World Scientific)
Krauss L M and Romanelli P 1990 *Astrophys. J.* **358** 47
Krauss L M, Terning J and Appelquist T 1993 *Phys. Rev. Lett.* **71** 823
Krauss L M, Guth A H, Spergel D N, Field G B and Press W H 1986 *Nature* **319** 748
Kundic T *et al* 1995 *Astrophys. J.* **455** L5
Kurki-Suonio H, Matzner R A, Olive K A and Schramm D N 1990 *Astrophys. J.* **353** 406
Kuzmin V A, Rubakov V A and Shaposhnikov M E 1985 *Phys. Lett.* **155B** 36
——1987 *Phys. Lett.* B**191** 171
Lahanas A B and Nanopoulos D V 1987 *Phys. Rep.* **145** 1
Lam W P and Ng K-W 1991 *Phys. Rev.* D**44** 3345
Landau L D and Lifshitz E M 1982 *Statistical Physics* (Oxford: Pergamon Press)
Lane K 1993 *The Building Blocks of Creation (TASI'93)* ed S Raby and T Walker (Singapore: World Scientific) p 381
Lang K R 1992 *Astrophysical Data: Planets and Stars* (Berlin: Springer-Verlag)
Langacker P 1981 *Phys. Rep.* **72** 185
——1988 *Neutrino Physics* ed H V Klapdor (Berlin: Springer-Verlag) p 71
——(ed) 1994 *Precision Tests of the Standard Electroweak Model* (Singapore: World Scientific)
Langacker P and Luo M 1991 *Phys. Rev.* D**44** 817
Langacker P, Segrè G and Soni S 1982 *Phys. Rev.* D**26** 3425
Langacker P, Sathiapalan B and Steigman G 1986 *Nucl. Phys.* B**266** 669
Langacker P, Petcov S T, Steigman G and Toshev S 1987 *Nucl. Phys.* B**282** 589
Laurent C 1983 *Proc. ESO Workshop on Primordial Helium* ed P Shaver *et al* (Garching: ESO) p 335
Lazarides G, Schaefer R, Seckel D and Shafi Q 1990 *Nucl. Phys.* B**346** 193
Lee B W and Schrock R E 1977 *Phys. Rev.* D**16** 1444

Lee B W and Weinberg S 1977 *Phys. Rev. Lett.* **39** 165
Leigh R G and Rattazzi R 1995 *Phys. Lett.* B**352** 20
LEP Electroweak Working Group 1995 *Preprint* CERN PPE/95-172
Liddle A and Lyth 1993 *Phys. Rep.* **231** 1
Linde A D 1979 *Rep. Prog. Phys.* **42** 389
——1984 *Rep. Prog. Phys.* **47** 925
——1990 *Particle Physics and Inflationary Cosmology* (New York: Harwood Academic)
——1996 *Phys. Rev.* D**53** R4129
Lindley D 1979 *Mon. Not. Roy. Astron. Soc.* **188** 15p
——1985 *Astrophys. J.* **294** 1
Linsky J L *et al* 1993 *Astrophys. J.* **402** 694
——1995 *Astrophys. J.* **451** 335
Longair M S 1981 *High Energy Astrophysics* (Cambridge: Cambridge University Press)
Lopez J L and Nanopoulos D V 1990 *Phys. Lett.* B**241** 392
Lynden-Bell D and Gilmore G (eds) 1990 *Baryonic Dark Matter* (Dordrecht: Kluwer)
Lyth D H and Stewart E D 1995 *Phys. Rev. Lett.* **75** 201
——1996 *Phys. Rev.* D**53** 1784
Lyttleton R A and Bondi H 1959 *Proc. R. Soc.* A**252** 313
Maalampi J and Roos M 1990 *Phys. Rep.* **186** 53
Mahajan S 1986 *Phys. Rev.* D**33** 338
Malaney R A and Fowler W A 1988 *Astrophys. J.* **333** 14
Malaney R A and Mathews G J 1993 *Phys. Rep.* **229** 145
Mampe W, Ageron P, Bates C, Pendlebury J M and Steyerl A 1989 *Phys. Rev. Lett.* **63** 593
Manohar A 1987 *Phys. Lett.* B**186** 370
Marciano W J 1991 *Annu. Rev. Nucl. Part. Sci.* **41** 469
Marciano W J and Sanda A 1977 *Phys. Lett.* **67B** 303
Massó E and Pomarol A 1989 *Phys. Rev.* D**40** 2519
Massó E and Toldrà R 1994a *Phys. Lett.* B**333** 132
——1994b *Phys. Rev.* D**52** 1755
Mather J C *et al* (COBE collab.) 1994 *Astrophys. J.* **420** 439
Mathews G J, Alcock C R and Fuller G M 1990a *Astrophys. J.* **349** 449
Mathews G J, Meyer B M, Alcock C and Fuller G M 1990b *Astrophys. J.* **358** 36
Mathews G J, Boyd R N and Fuller G M 1993 *Astrophys. J.* **403** 65
McCullough P R 1992 *Astrophys. J.* **390** 213
McKellar B H J and Pakvasa S 1983 *Phys. Lett.* **122B** 33
McLerran L 1986 *Rev. Mod. Phys.* **58** 1021
Melnick J, Heydani-Malyeri M and Leisy P 1992 *Astron. Astrophys.* **253** 16
Michaud G and Charbonneau P 1991 *Space Sci. Rev.* **57** 1
Mihalas D and Binney J 1981 *Galactic Astronomy* (San Francisco: Freeman)
Milne E A 1935 *Relativity, Gravitation and World Structure* (Oxford: Clarendon Press)
Miyama S and Sato K 1978 *Prog. Theo. Phys.* **60** 1703
Mohapatra R N and Marshak R 1980 *Phys. Rev. Lett.* **44** 1316
——1992 *Unification and Supersymmetry* (New York: Spinger-Verlag)
Mohapatra R N and Pal P B 1991 *Massive Neutrinos in Physics and Astrophysics* (Singapore: World

Scientific)
Molaro P, Primas F and Bonifacio P 1995 *Astron. Astrophys.* **295** L47
Morgan J A 1981a *Phys. Lett.* **102B**247
——1981b *Mon. Not. Roy. Astron. Soc.* **195** 173
Moroi T, Murayama H and Yamaguchi M 1993 *Phys. Lett.* B**303** 289
Mukhanov V F, Feldman H A and Brandenberger R H 1982 *Phys. Rep.* **215** 203
Nanopoulos D V (ed) 1991 *Proc. Intern. School of Astroparticle Physics* (Singapore: World Scientific)
Nanopoulos D V, Olive K A and Srednicki M 1983 *Phys. Lett.* **127B** 30
Nath P, Arnowitt R and Chamseddine A 1984 *Applied N=1 Supergravity* (Singapore: World Scientific)
Nelson A E and Seiberg N 1994 *Nucl. Phys.* B**416** 46
Nieves J F 1983 *Phys. Rev.* D**28** 1664
Nilles H P 1984 *Phys. Rep.* **110** 1
——1990 *Int. J. Mod. Phys.* A**5** 4199
Nilles H P, Srednicki M and Wyler D 1983 *Phys. Lett.* **120B** 346
Nilsson J S, Gustafson B and Skagerstam B -S (ed) 1991 *The Birth and Early Evolution of Our Universe* (Singapore: World Scientific)
Norris J E, Ryan S G and Stringfellow G S 1994 *Astrophys. J.* **423** 386
Nötzold D and Raffelt G 1988 *Nucl. Phys.* B**307** 924
Nussinov S, 1985 *Phys. Lett.* **165B** 55
Oberauer L and von Feilitzsch F 1992 *Rep. Prog. Phys.* **55** 1093
Oberauer L and von Feilitzsch F and Mössbauer R L 1987 *Phys. Lett.* B**198** 113
Olive K A 1990a *Phys. Rep.* **190** 309
——1990b *Science* **251** 1194
Olive K A and Schramm D N 1992 *Nature* **360** 439
Olive K A and Scully S T 1996 *Int. J. Mod. Phys.* A**11** 409
Olive K A and Steigman G 1995a *Astrophys. J. Suppl.* **97** 49
——1995b *Phys. Lett.* B**354** 357
Olive K A and Thomas D 1991 *Nucl. Phys.* B**335** 192
Olive K A and Turner M S 1982 *Phys. Rev.* D**25** 213
Olive K A, Schramm D N and Steigman G 1981a *Nucl. Phys.* B**180** 497
Olive K A, Schramm D N, Steigman G, Turner M S and Yang J 1981b *Astrophys. J.* **246** 557
Olive K A, Schramm D N, Steigman G and Walker T P 1990 *Phys. Lett.* B**236** 454
Olive K A, Schramm D N, Thomas D and Walker T P 1991 *Phys. Lett.* **65B** 239
Olive K A, Rood R T, Schramm D N, Truran J and Vangioni-Flam E 1995 *Astrophys. J.* **444** 680
Ostriker J, Thompson C and Witten E 1986 *Phys. Lett.* B**180** 231
Paczyński B 1986 *Astrophys. J.* **304** 1
Padmanabhan T 1993 *Structure Formation in the Universe* (Cambridge: Cambridge University Press)
Padmanabhan T and Vasanti R 1982 *Phys. Lett.* **89A** 327
Pagel B E J 1982 *Proc. R. Soc.* A**307** 19
——1987 *An Unified View of the Macro- and Micro-Cosmos* ed A De Rújula *et al* (Singapore: World Scientific) p 399
——1992 *Observational and Physical Cosmology* ed F Sanchez *et al* (Cambridge: Cambridge University Press) p 116
——1993 *Proc. Nat. Acad. Sci.* **90** 4789

Pagel B E J and Kazlaukas A 1992 *Mon. Not. Roy. Astron. Soc.* **256** 49p

Pagel B E J, Simonson E A, Terlevich R J and Edmunds M G 1992 *Mon. Not. Roy. Astron. Soc.* **255** 325

Pagels H and Primack J R 1982 *Phys. Rev. Lett.* **48** 223

Pal P B 1983 *Nucl. Phys.* B**277** 237

——1992 *Int. J. Mod. Phys.* A**7** 5387

Pal P B and Wolfenstein L 1982 *Phys. Rev.* D**25** 766

Palla F, Galli D and Silk J 1995 *Astrophys. J.* **451** 44

Particle Data Group 1990 *Phys. Lett.* B**239** 1

——1992 *Phys. Rev.* D**45** S1

——1994 *Phys. Rev.* D**50** 1173

——1996 *Phys. Rev.* D**54** 1

Pasachoff J M and Vidal-Madjar A 1989 *Comm. Astrophys.* **14** 61

Peacock J A, Heavens A F and Davies A T (ed) 1990 *Physics of the Early Universe* (Bristol: Adam Hilger)

Peccei R D 1989 *CP Violation* ed C Jarlskog (Singapore: World Scientific) p 503

Peebles P J E 1966a *Phys. Rev. Lett.* **16** 411

——1966b *Astrophys. J.* **146** 542

——1971 *Physical Cosmology* (Princeton: Princeton University Press)

——1980 *The Large-Scale Structure of the Universe* (Princeton: Princeton University Press)

——1987 *Astrophys. J.* **315** L73

——1993 *Principles of Physical Cosmology* (Princeton: Princeton University Press)

Peimbert M and Torres-Peimbert S 1974 *Astrophys. J.* **193** 327

Penrose R 1979 *General Relativity: An Einstein Centenary Survey* ed S W Hawking and W Israel (Cambridge: Cambridge University Press) p 581

——1989 *Proc. 14th Texas Symp. on Relativistic Astrophys.* ed E Fenyves (New York: New York Academy of Sciences) p 249

Perkins D 1984 *Annu. Rev. Nucl. Part. Sci.* **34** 1

——1993 *Nucl. Phys.* B**399** 3

Persic M and Salucci P 1992 *Mon. Not. Roy. Astron. Soc.* **258** 14p

Petcov S 1977 *Sov. J. Nucl. Phys.* **25** 344 (erratum **25** 698)

Philipsen O, Teper M and Wittig H 1996 *Nucl. Phys.* B**B469** 445

Pinsonneault M H, Deliyannis C P and Demarque P 1992 *Astrophys. J. Suppl.* **78** 179

Piran T and Weinberg S (ed) 1986 *Intersection between Particle Physics and Cosmology: Proc. 1st Jerusalem Winter School for Theoretical Physics* (Singapore: World Scientific)

Plümacher M 1996 *Preprint* hep-ph/9604229

Polchinski J 1996 *Preprint* NSF-ITP-96-60 (hep-th/9607050)

Prantzos N, Cassé and Vangioni-Flam E 1993 *Astrophys. J.* **403** 630

Preskill J 1984 *Annu. Rev. Nucl. Part. Sci.* **34** 461

Primack J, Seckel D and Sadoulet B 1988 *Annu. Rev. Nucl. Part. Sci.* **38** 751

Protheroe R J, Stanev T and Berezinsky V S 1995 *Phys. Rev.* D**51** 4134

Quirós M 1991 *Phys. Lett.* B**267** 27

Raffelt G 1990 *Phys. Rep.* **198** 1

Rai B and Senjanović G 1994 *Phys. Rev.* D**49** 2729

Rana N C 1991 *Annu. Rev. Astron. Astrophys.* **29** 129

Rana N C and Mitra B 1991 *Phys. Rev.* D**44** 393

Randall L and Thomas S 1995 *Nucl. Phys.* B**449** 229

Rangarajan R 1995a *Nucl. Phys.* B**454** 357

——1995b *Nucl. Phys.* B**454** 369

Rauscher T, Applegate J H, Cowan J J, Thielemann F-K and Wiescher M 1994 *Astrophys. J.* **429** 499

Rebolo R, Molaro P and Beckman J E 1988 *Astron. Astrophys.* **192** 192

Reeves H 1991 *Phys. Rep.* **201** 335

——1994 *Rev. Mod. Phys.* **66** 193

Reeves H, Audouze J, Fowler W A and Schramm D N 1973 *Astrophys. J.* **179** 909

Reno M H and Seckel D 1988 *Phys. Rev.* D**37** 3441

Rephaeli Y 1990 *The Cosmic Microwave Background: 25 Years Later* ed N Mandolesi and N Vittorio (Dordrecht: Kluwer Academic) p 67

Rich J, Lloyd-Owen D, and Spiro M 1987 *Phys. Rep.* **151** 239

Riley S P and Irvine J M 1991 *J. Phys. G: Nucl. Phys.* **17** 35

Rindler W 1977 *Essential Relativity* (Berlin: Springer-Verlag)

Rohm R 1984 *Nucl. Phys.* B**237** 553

Romão J C and Freire F (ed) 1994 *Electroweak Physics and the Early Universe* (London: Plenum Press)

Roncadelli M and Senjanović G 1981 *Phys. Lett.* **107B** 59

Rood R T, Steigman G and Tinsley B 1976 *Astrophys. J.* **207** L57

Rood R T, Bania T M and Wilson T L 1992 *Nature* **355** 618

Ross G G 1984 *Grand Unified Theories* (New York: Benjamin/Cummings)

Ross G G and Roberts R G 1992 *Nucl. Phys.* B**377** 571

Ross G G and Sarkar S 1996 *Nucl. Phys.* B**461** 597

Rowan-Robinson M 1985 *The Cosmological Distance Ladder* (New York: Freeman)

——1988 *Space Sci. Rev.* **48** 1

Rubakov V A and Shaposhnikov M E 1996 *Preprint* CERN-TH/96-13 (hep-ph/9603208) [*Sov. Phys. Uspekhi*, in press]

Rugers M and Hogan C J 1996a *Astrophys. J.* **459** L1

——1996b *Astron. J.* **111** 2135

Ryan S G, Beers T C, Deliyannis C P and Thorburn J 1996 *Astrophys. J.* **458** 543

Sadoulet B 1992 *Research Directions for the Decade: Proc. 1990 Summer Study on High-Energy Physics, Snowmass* ed E L Berger (Singapore: World Scientific) p 3

Sakharov A D 1967 *JETP Lett.* **5** 24

Salati P, Delbourgo-Salvador P and Audouze J 1987 *Astron. Astrophys.* **173** 1

Sanchez N and Zichichi A (ed) 1992 *Current Topics in Astrofundamental Physics* (Singapore: World Scientific)

Sarkar S 1985 *Superstrings, Supergravity and Unified Theories* ed G Furlan *et al* (Singapore: World-Scientific) p 465

——1986 *Proc. XXVI Intern. Symp. on Multiparticle Dynamics* ed M Markatyan *et al* (Singapore: World Scientific) p 863

——1988 *Astrophys. Lett. Comm.* **27** 293

——1991 *Observational Tests of Cosmological Inflation* ed T Shanks *et al* (Dordrecht: Kluwer Academic) p 91

——1996 *Proc. Intern. EPS Conf. on High Energy Physics, Brussels* ed J Lemonne *et al* (Singapore: World Scientific) p 95

——1997 *The Big Bang: A Laboratory for Particle Physics* (Cambridge: Cambridge University Press) to appear

Sarkar S and Cooper A M 1984 *Phys. Lett.* **148B** 347

Sasselov D and Goldwirth D S 1995 *Astrophys. J.* **444** L5

Sato K and Audouze J (ed) 1991 *Primordial Nucleosynthesis and Evolution of Early Universe: Proc. Intern. Conf., Tokyo* ((Dordrecht: Kluwer Academic)

Sato K and Kobayashi M 1977 *Prog. Theo. Phys.* **58** 1775

Satz H 1985 *Annu. Rev. Nucl. Part. Sci.* **35** 245

Savage M J, Malaney R A and Fuller G M 1991 *Astrophys. J.* **368** 1

Scalo J M 1986 *Fund. Cosm. Phys.* **11** 1

Schechter J and Valle J W F 1982 *Phys. Rev.* D**25** 774

Scherrer R J 1983 *Mon. Not. Roy. Astron. Soc.* **205** 683

——1984 *Mon. Not. Roy. Astron. Soc.* **210** 359

Scherrer R J and Turner M S 1985 *Phys. Rev.* D**31** 681

——1986 *Phys. Rev.* **D33** 1585 (erratum D**34** 3263)

——1988a *Astrophys. J.* **331** 19

——1988b *Astrophys. J.* **331** 33

Scherrer R J, Cline J, Raby S and Seckel D 1991 *Phys. Rev.* D**44** 3760

Schramm D N and Wagoner R V 1977 *Annu. Rev. Nucl. Sci.* **27** 37

Schreckenbach K and Mampe W 1992 *J. Phys. G: Nucl. Phys.* **18** 1

Schrock R E 1981 *Phys. Rev.* D**24** 1232,1275

Sciama D W 1982 *Phys. Lett.* **118B** 327

——1993 *Modern Cosmology and the Dark Matter Problem* (Cambridge: Cambridge University Press)

Scully S T, Cassé M, Olive K A, Schramm D N, Truran J and Vangioni-Flam E 1996 *Astrophys. J.* **462** 960

Seckel D 1993 *Preprint* BA-93-16 (hep-ph/9305311)

Sengupta S and Pal P B 1996 *Phys. Lett.* B**365** 175

Senjanović G 1986 *Inner Space–Outer Space* eds E W Kolb *et al* (Chicago: University of Chicago Press) p 554

Setti G and Van Hove L (ed) 1984 *First ESO-CERN Symp.: Large-scale Structure of the Universe, Cosmology and Fundamental Physics* (Geneva: CERN)

Shapiro P and Wasserman I 1981 *Nature* **289** 657

Shapiro P, Teukolsky S A and Wasserman I 1980 *Phys. Rev. Lett.* **45** 669

Shaposhnikov M E 1991 *Phys. Scr.* T**36** 183

——1992 *Electroweak Interactions and Unified Theories* ed J Tran Thanh Van (Gif Sur Yvette: Editions Frontiéres) p 201

Shaver P A, Kunth D and Kjär K (ed) 1983 *Primordial Helium* (Garching: European Southern Observatory)

Sher M 1989 *Phys. Rep.* **179** 273

Shi X, Schramm D N and Fields B 1993 *Phys. Rev.* D**48** 2563

Shields G 1987 *13th Texas Symp. on Relativistic Astrophysics* ed P Ulmer (Singapore: World Scientific Press) p 192

Shu F H 1981 *The Physical Universe: An Introduction to Astronomy* (Mill Valley, CA: University Science Books)

Shuryak E 1980 *Phys. Rep.* **61** 2

Shvartsman V F 1969 *JETP Lett.* **9** 184

Sigl G, Jedamzik K, Schramm D N and Berezinsky V S 1995 *Phys. Rev.* D**52** 6682

Simpson J J 1985 *Phys. Rev. Lett.* **54** 1891

Skillman E and Kennicutt R C 1993 *Astrophys. J.* **411** 655

Skillman E, Terlevich R J, Kennicutt R C, Garnett D R and Terlevich E 1993 *Ann. N. Y. Acad. Sci.* **688** 739

Smilga A V 1995 *Preprint* SUNY-NTG-95-304 (hep-ph/9508305)

Smith M S, Kawano L H and Malaney R A 1993 *Astrophys. J. Suppl.* **85** 219

Smith P F 1988 *Contemp. Phys.* **29** 159

Smith P F and Lewin J D 1990 *Phys. Rep.* **187** 203

Smith P F *et al* 1982 *Nucl. Phys.* B**206** 333

Smith V V, Lambert D L and Nissen P E 1992 *Astrophys. J.* **408** 262

Smits D P 1991 *Mon. Not. Roy. Astron. Soc.* **248** 20

——1996 *Mon. Not. Roy. Astron. Soc.* **278** 683

Smoot G F *et al* (COBE collab.) 1992 *Astrophys. J.* **396** L1

Songaila A, Cowie L L, Hogan C J and Rugers M 1994 *Nature* **368** 599

Spite F and Spite M 1982 *Astron. Astrophys.* **115** 357

Spite M, Spite F, Peterson R C and Chafee F C 1987 *Astron. Astrophys.* **172** L9

Srednicki M (ed) 1990 *Particle Physics and Cosmology: Dark Matter* (Amsterdam: North-Holland)

Srednicki M, Watkins W and Olive K A 1988 *Nucl. Phys.* B**310** 693

Starkman G D 1992 *Phys. Rev.* D**45** 476

Starobinsky, A A 1993 *JETP Lett.* **57** 622

Steigman G 1976 *Annu. Rev. Astron. Astrophys.* **14** 33

——1979 *Annu. Rev. Nucl. Part. Sci.* **29** 313

Steigman G and Tosi M 1992 *Astrophys. J.* **401** 150

——1995 *Astrophys. J.* **453** 173

Steigman G, Schramm D N and Gunn J 1977 *Phys. Lett.* **66B** 202

Steigman G, Olive K A and Schramm D N 1979 *Phys. Rev. Lett.* **43** 239

Steigman G, Olive K A, Schramm D N and Turner M S 1986 *Phys. Lett.* B**176** 33

Steigman G, Fields B D, Olive K A, Schramm D N and Walker T P 1993 *Astrophys. J.* **415** L35

Steinhardt P J 1995 *Int. J. Mod. Phys.* A**10** 1091

Stevens D, Scott D and Silk J 1993 *Phys. Rev. Lett.* **71** 20

Sugiyama N and Silk J 1994 *Phys. Rev. Lett.* **73** 509

Szalay A S and Marx G 1976 *Astron. Astrophys.* **49** 437

Terasawa N and Sato K 1987 *Phys. Lett.* B**185** 412

——1988 *Prog. Theo. Phys.* **80** 468

——1991 *Phys. Scr.* T**36** 60

Terasawa N, Kawasaki M and Sato K 1988 *Nucl. Phys.* B**302** 697

Thomas D, Schramm D N, Olive K A and Fields B D 1993 *Astrophys. J.* **406** 569

Thomas S 1995 *Phys. Lett.* B**351** 424

Thorburn J A 1994 *Astrophys. J.* **421** 318

Thuan T X, Izotov Y I and Lipovetsky V A 1996 *Interplay between Massive Star Formation, the ISM and Galaxy Evolution* ed D. Kunth *et al* (Gif-Sur-Yvette: Edition Frontiéres)

Tinsley B M 1980 *Fund. Cosm. Phys.* **5** 287

Toussaint D 1992 *Nucl. Phys. B (Proc. Suppl.)* **26** 3

Trimble V 1989 *Annu. Rev. Astron. Astrophys.* **25** 425

Turner M S 1990 *Phys. Rep.* **197** 67

——1991 *Phys. Scr.* T**36** 167

——1993 *Proc. Nat. Acad. Sci.* **90** 4827

Tytler D and Burles S 1996 *Preprint* astro-ph/9606110

Tytler D, Fan X and Burles S 1996 *Nature* **381** 207

Unruh W G and Semenoff G W (ed) 1988 *The Early Universe* (Dordrecht: Reidel)

Valle J W F 1991 *Prog. Part. Nucl. Phys.* **26** 91

Van den Bergh S 1992 *Pub. Astron. Soc. Pacific* **104** 861

——1994 *Pub. Astron. Soc. Pacific* **106** 1113

Van Nieuwenhuizen P 1981 *Phys. Rep.* **68** 189

Vangioni-Flam E and Casseé M 1995 *Astrophys. J.* **441** 471

Vangioni-Flam E, Olive K A and Prantzos N 1994 *Astrophys. J.* **427** 618

Vauclair S 1988 *Astrophys. J.* **335** 971

Vauclair S and Charbonnel C 1995 *Astron. Astrophys.* **295** 715

Verkerk P *et al* 1992 *Phys. Rev. Lett.* **68** 1117

Vidal-Madjar A 1986 *Space Astronomy and Solar System Exploration* [ESA SP-268] (Noordwijk: ESA) p 73

Vilenkin A 1985 *Phys. Rep.* **121** 263

Vilenkin A and Shellard E P S 1994 *Cosmic Strings and Other Topological Defects* (Cambridge: Cambridge University Press)

Vysotskiĭ M I, Dolgov A D and Zel'dovich Ya B 1977 *JETP Lett.* **26** 188

Wagoner R V 1969 *Astrophys. J. Suppl.* **18** 247

——1973 *Astrophys. J.* **179** 343

——1980 *Les Houches Session XXXII: Physical Cosmology* ed R Balian *et al* (Amsterdam: North-Holland) p 395

——1990 *Modern Cosmology in Retrospect* ed R Bertotti *et al* (Cambridge: Cambridge University Press) p 159

Wagoner R V, Fowler W A and Hoyle F 1967 *Astrophys. J.* **148** 3

Walker T P, Steigman G, Schramm D N, Olive K A and Kang H-S 1991 *Astrophys. J.* **376** 51

Wampler E J *et al* 1995 *Preprint* astro-ph/9512084 [*Astron. Astrophys.*, in press]

Wasserburg G J, Boothroyd A I and Sackmann I 1995 *Astrophys. J.* **447** L37

Webb J K, Carswell R F, Irwin M J and Penston M V 1991 *Mon. Not. Roy. Astron. Soc.* **250** 657

Weinberg S 1972 *Gravitation and Cosmology* (New York: Wiley)

——1977 *The First Three Minutes* (New York: Basic Books)

——1980 *Phys. Scr.* **21** 773

——1982 *Phys. Rev. Lett.* **48** 1303

——1989 *Rev. Mod. Phys.* **61** 1

Wess J and Bagger J 1993 *Supersymmetry and Supergravity* (Princeton: Princeton University Press)

White M, Viana P T P, Liddle A R and Scott D 1996 *Preprint* SUSSEX-AST 96/5-2 (astro-ph/9605057)

[submitted to *Mon. Not. Roy. Astron. Soc.*]

White S D M 1994 *Preprint* astro-ph/9410043 [to appear in *Les Houches Session 60: Cosmology and Large Scale Structure*

White D A and Fabian A 1995 *Mon. Not. Roy. Astron. Soc.* **273** 72

White S D M, Navarro J F, Evrad A E and Frenk C S 1993 *Nature* **366** 429

Wilczek F 1991 *Phys. Scr.* T**36** 281

Wilkinson D H 1982 *Nucl. Phys.* A**377** 474

Wilson T L and Rood R T 1994 *Annu. Rev. Astron. Astrophys.* **32** 191

Witten E 1981a *Comm. Math. Phys.* **80** 381

——1981b *Nucl. Phys.* B**188** 513

——1984 *Phys. Rev.* D**30** 272

——1985a *Nucl. Phys.* B**249** 557

——1985b *Nucl. Phys.* B**258** 75

Wolfram S 1979 *Phys. Lett.* **82B** 65

Wright E L *et al* 1994 *Astrophys. J.* **420** 450

Yaffe L G 1995 *Preprint* hep-ph/9512265

Yahil A and Beaudet G 1976 *Astrophys. J.* **206** 26

Yamazaki T *et al* 1984 *Proc. Neutrino '84* eds K Kleinknecht and E A Paschos (Singapore: World Scientific Press) p 183

Yang J, Schramm D N, Steigman G and Rood R T 1979 *Astrophys. J.* **227** 697

Yang J, Turner M S, Steigman G, Schramm D N and Olive K A 1984 *Astrophys. J.* **281** 493

Yoshimura M, Totsuka Y, Nakamura K and Lim C S (ed) 1988 *Proc. 2nd Workshop on Elementary Particle Picture of the Universe* (Tsukuba: KEK)

Zdziarski A 1988 *Astrophys. J.* **335** 786

Zdziarski A and Svensson R 1989 *Astrophys. J.* **344** 551

Zee A (ed) 1982 *Unity of Forces in the Universe* (Singapore: World Scientific)

Zel'dovich Ya B 1965 *Advances in Astron. and Astrophys.* **3** 241

Zel'dovich Ya B, Kobzarev I Yu and Okun L B 1975 *Sov. Phys. JETP* **40** 1

Zwirner F 1996 *Proc. Intern. EPS Conf. on High Energy Physics, Brussels* ed J Lemonne *et al* (Singapore: World Scientific) p 743

Tables and table captions

Table 1. Thermodynamic history of the RD era

T	Threshold (GeV)	Particle Content	$g_{\mathrm{R}}(T)$	$\frac{N_\gamma(T_0)}{N_\gamma(T)}$
$< m_e$	0.511×10^{-3}	γ (+ 3 decoupled ν's)	2	1
$m_e - T_{\mathrm{D}}(\nu)$	#	add $e^{\pm}$	11/2	2.75
$T_{\mathrm{D}}(\nu) - m_\mu$	0.106	ν's become interacting	43/4	2.75
$m_\mu - m_\pi$	0.135	add $\mu^{\pm}$	57/4	3.65
$m_\pi - T_{\mathrm{c}}^{\mathrm{qh}}$	$	add $\pi^{\pm}$, π^0	69/4	4.41
$T_{\mathrm{c}}^{\mathrm{qh}} - m_s$	0.194	γ, 3ν's, $e^{\pm}$, $\mu^{\pm}$	205/4	13.1
		u, $\bar{u}$, d, $\bar{d}$, 8 g's		
$m_s - m_c$	1.27 ± 0.05	add s, $\bar{s}$	247/4	15.8
$m_c - m_\tau$	1.78	add c, $\bar{c}$	289/4	18.5
$m_\tau - m_b$	4.25 ± 0.10	add $\tau^{\pm}$	303/4	19.4
$m_b - m_W$	80.3 ± 0.3	add b, $\bar{b}$	345/4	22.1
$m_W - m_t$	180 ± 12	add $W^{\pm}$, Z^0	381/4	24.4
$m_t - m_{H^0}$	†	add t, $\bar{t}$	423/4	27.1
$m_{H^0} - T_{\mathrm{c}}^{\mathrm{EW}}$	‡	add H^0	427/4	27.3

Neutrinos decouple from the thermal plasma at $T_{\mathrm{D}}(\nu) \approx 2.3 - 3.5\,\mathrm{MeV}$.

$ $T_{\mathrm{c}}^{\mathrm{qh}} \approx 150 - 400\,\mathrm{MeV}$ characterizes the quark-hadron phase transition (assumed to be adiabatic).

† We have assumed that the Higgs boson is heavier than both the $W^{\pm}, Z^0$ bosons and the t quark.

‡ Note that g_{R} does *not* change when the $SU(2)_{\mathrm{L}} \otimes U(1)_Y$ symmetry is restored at $T_{\mathrm{c}}^{\mathrm{EW}} \sim 300$ GeV since the total number of degrees of freedom in the gauge plus Higgs fields is invariant.

Figure captions

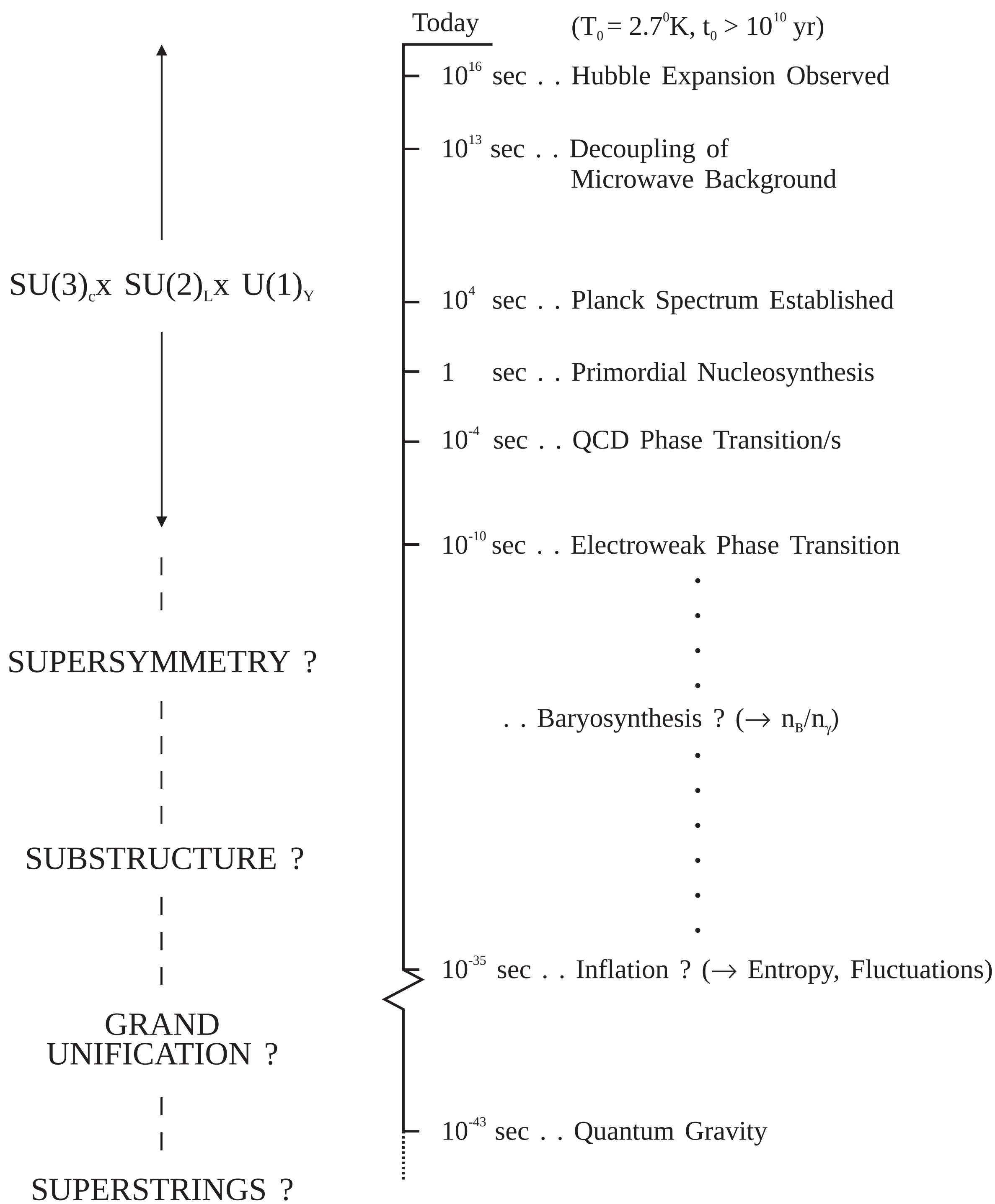

Figure 1. The cosmological history of the universe

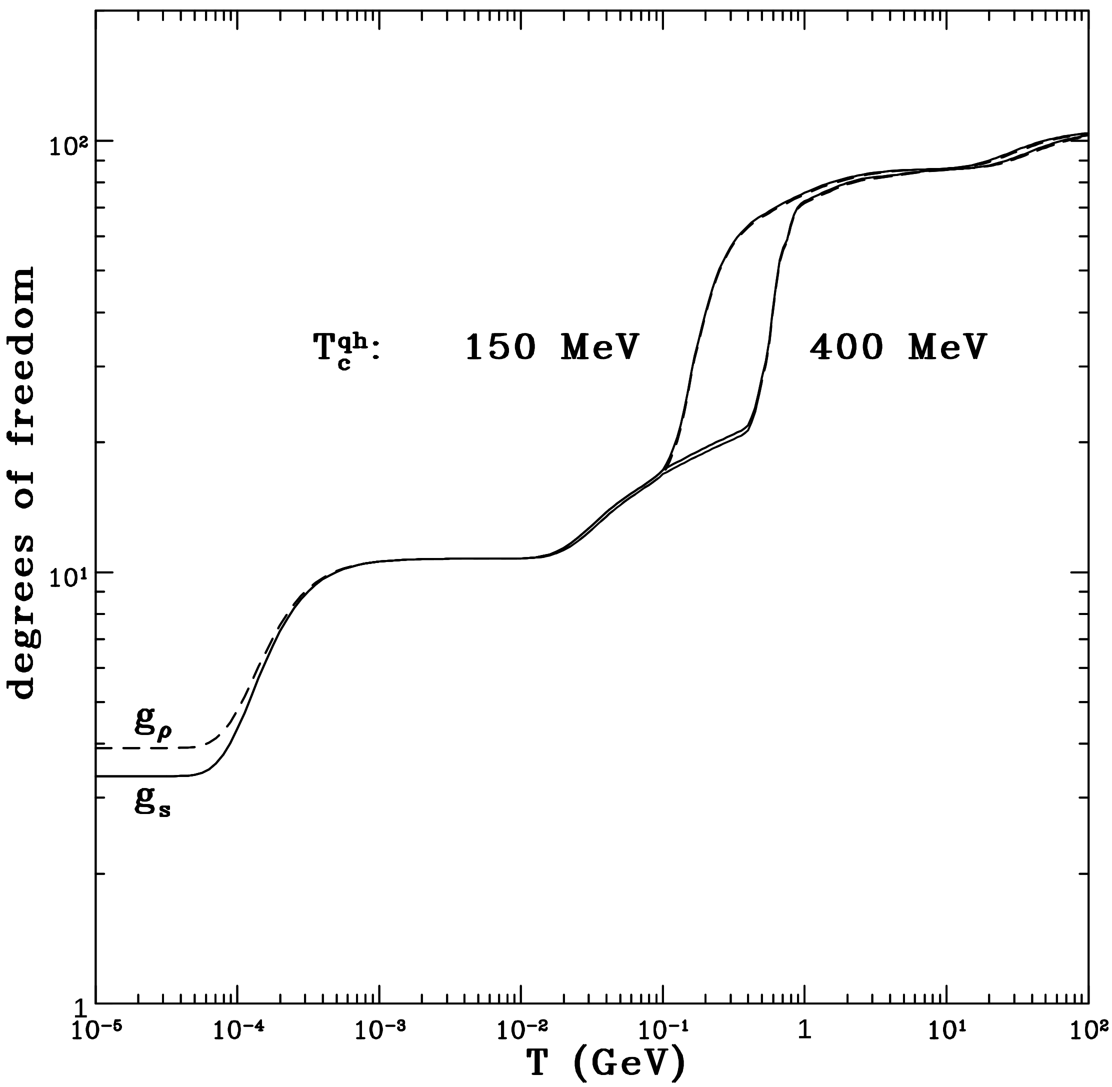

Figure 2. The number of relativistic degrees of freedom characterizing the entropy density g_s (dashed line) and the energy density g_ρ (solid line), as a function of temperature in the Standard $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ Model.

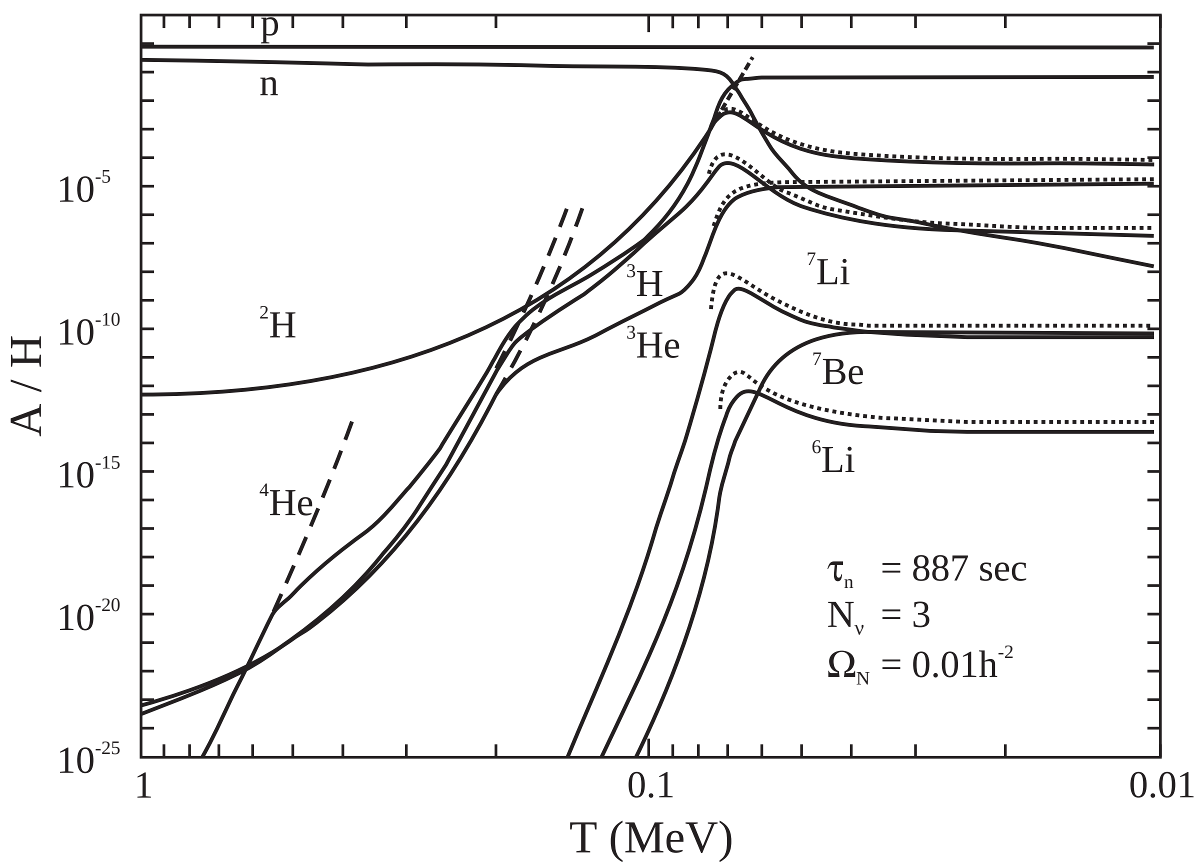

Figure 3. Evolution of the abundances of primordially synthesized light elements with temperature according to the Wagoner (1973) numerical code as upgraded by Kawano (1992). The dashed lines show the values in nuclear statistical equilibrium while the dotted lines are the 'freeze-out' values as calculated analytically by Esmailzadeh *et al* (1991).

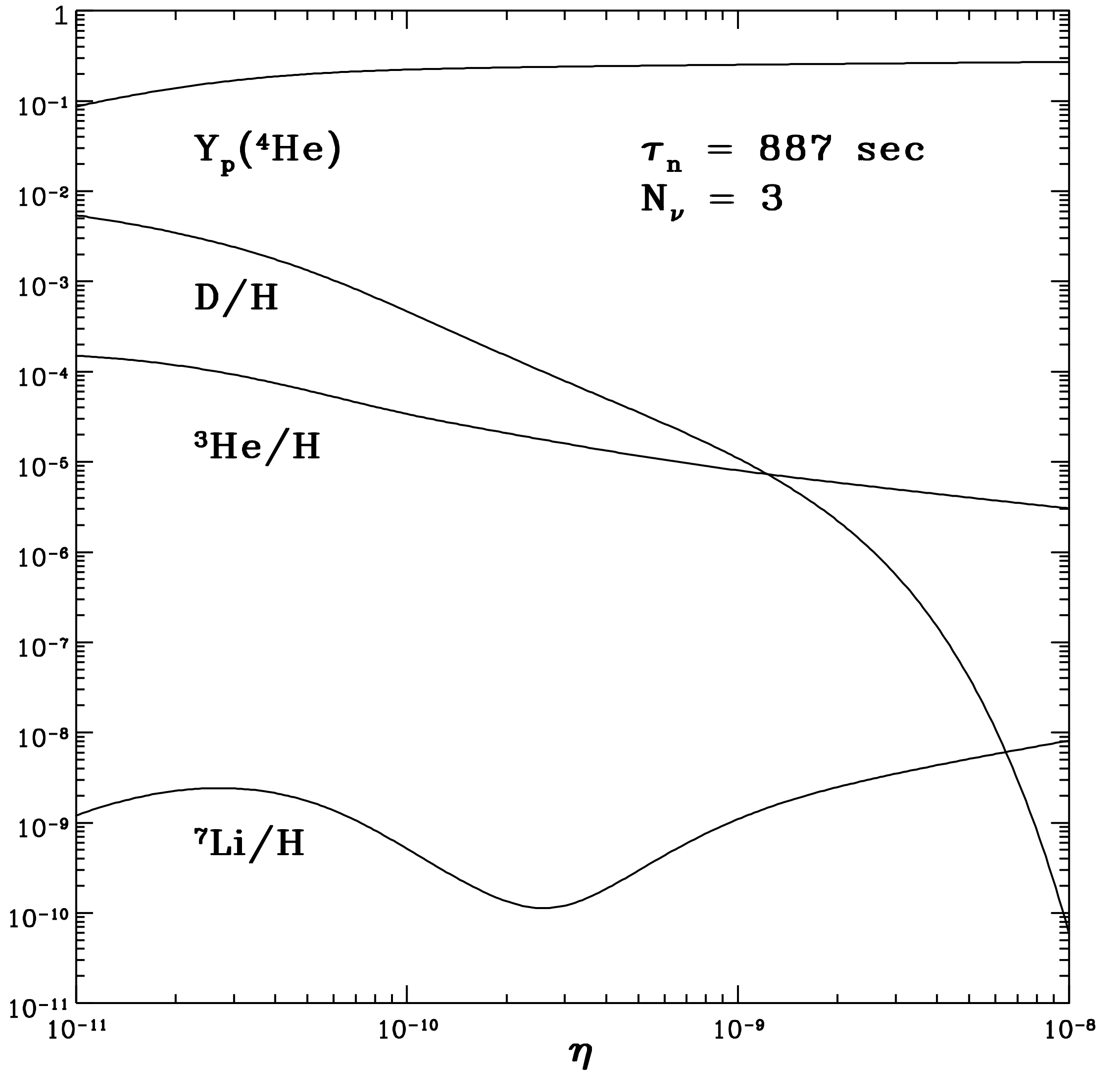

Figure 4. Dependence of primordially synthesized light element abundances on the nucleon-to-photon ratio η, calculated using the upgraded Wagoner code.

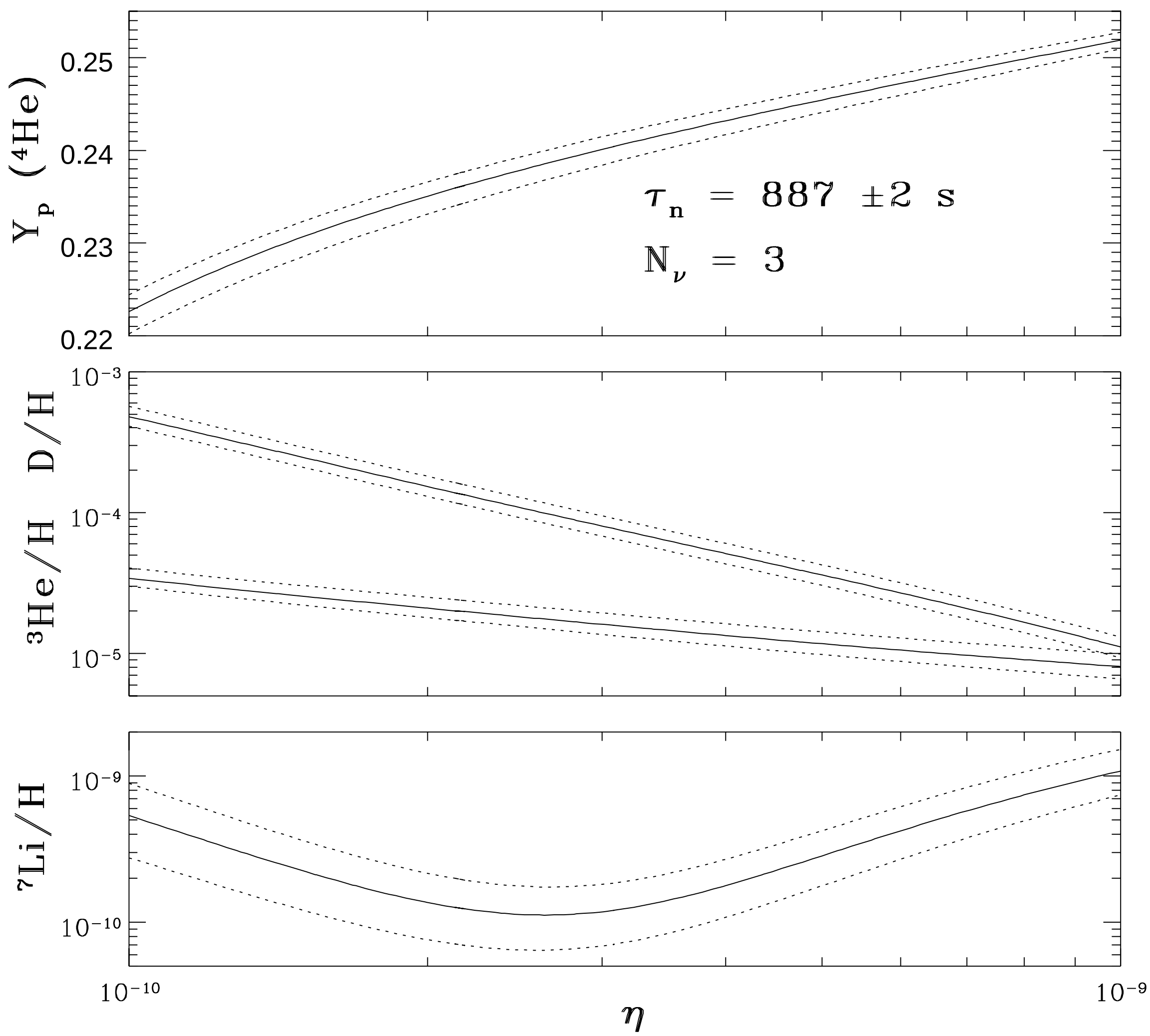

Figure 5. Monte Carlo results (Krauss and Kernan 1995) for the 95% c.l. limits (dashed lines) on primordially synthesized elemental abundances, along with their central values (full lines). Note that the ^{4}He mass fraction $Y_{\rm p}$ is shown on a linear scale.

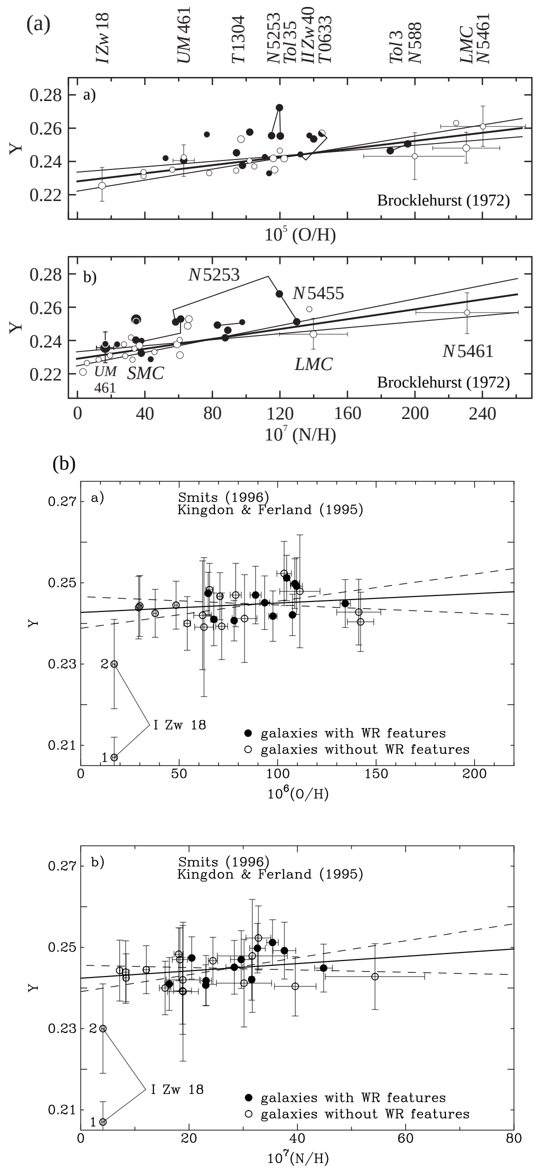

Figure 6. Regressions of the helium mass fraction against the oxygen and nitrogen abundances in extragalactic low-metallicity HII regions, with (filled circles) and without (open circles) broad Wolf-Rayet features. Panel (a) shows abundances for 33 objects obtained using the Brocklehurst (1972) emissivities by Pagel *et al* (1992), with the maximum-likelihood linear fits (with $\pm 1\sigma$ limits) for the latter category. Panel (b) shows abundances for 27 objects obtained using the Smits (1996) emissivities by Izotov *et al* (1996) along with the maximum-likelihood linear fits (with $\pm 1\sigma$ limits).

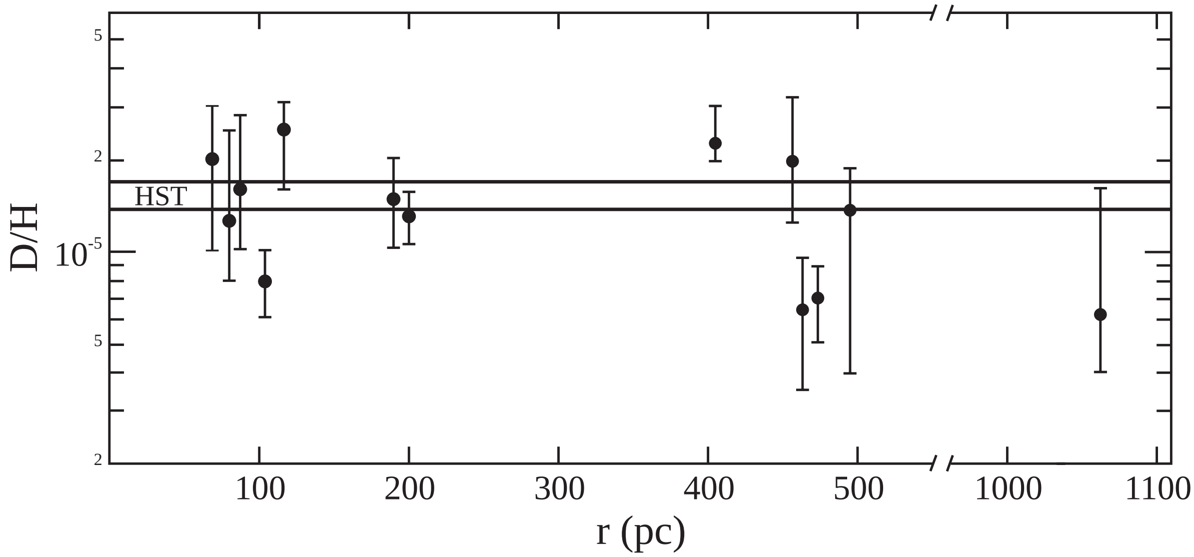

Figure 7. The interstellar deuterium abundance as observed by *Copernicus* and *IUE* towards distant hot stars. The band shows the value measured towards the nearby star *Capella* (at 12.5 kpc) by the *Hubble Space Telescope* (Linsky *et al* 1995).

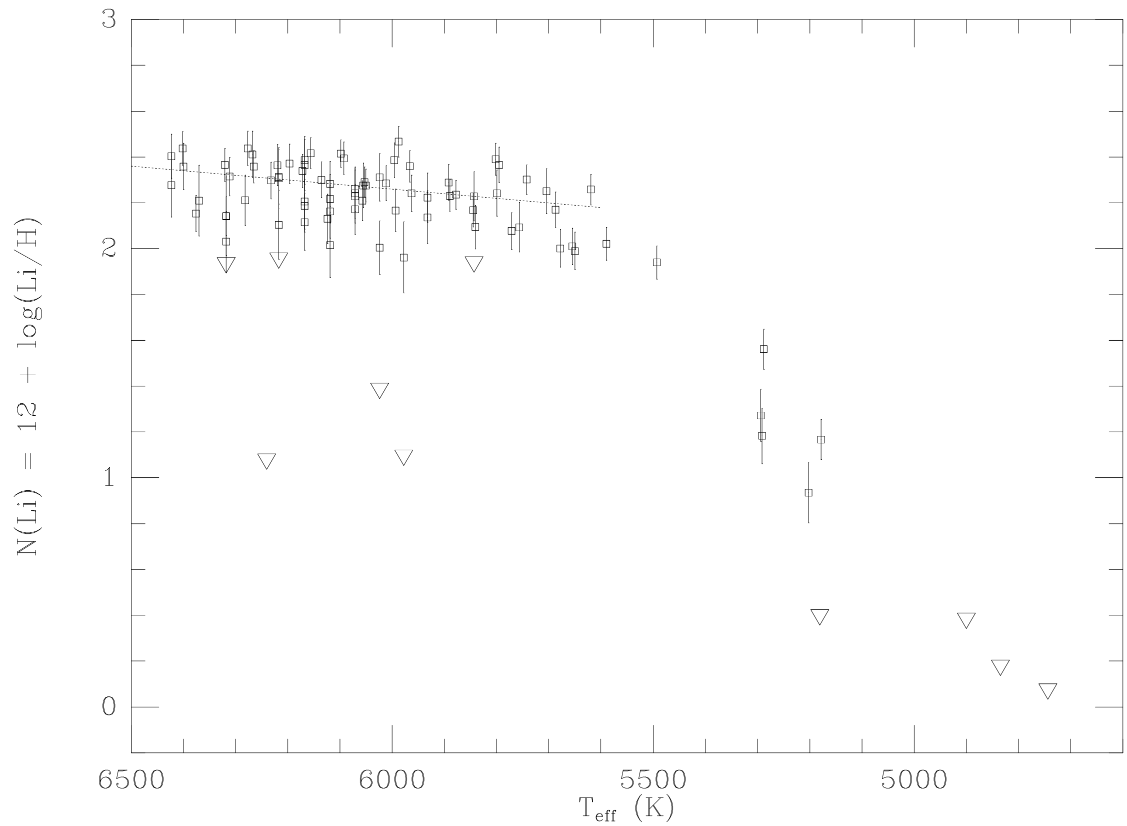

Figure 8. The Lithium abundance in 90 halo dwarf and main-sequence turnoff stars versus their effective surface temperature. Error bars indicate the 1σ interval for detections while triangles denote 3σ upper limits for non-detections; the dotted line is a fit which minimizes the absolute deviation of the detections (from Thorburn 1994).

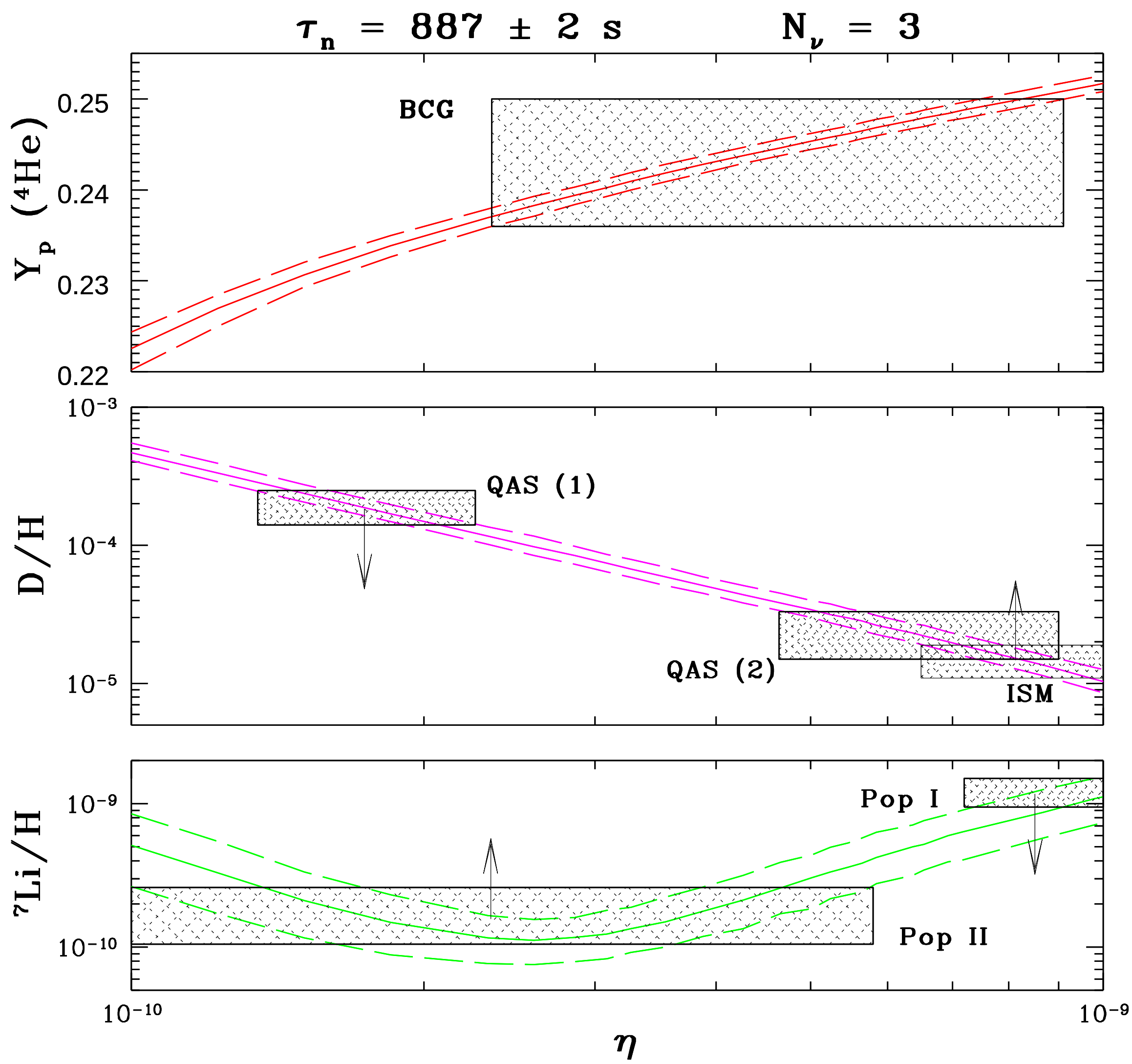

Figure 9. Concordance of the predicted abundances with present observational bounds (from Kernan and Sarkar 1996). Note that only the ^{4}He abundance inferred from BCG is established to be primordial. The two conflicting measurements of the D abundance in QAS are shown along with its ISM value, and both the Pop I and Pop II ^{7}Li abundances are shown.

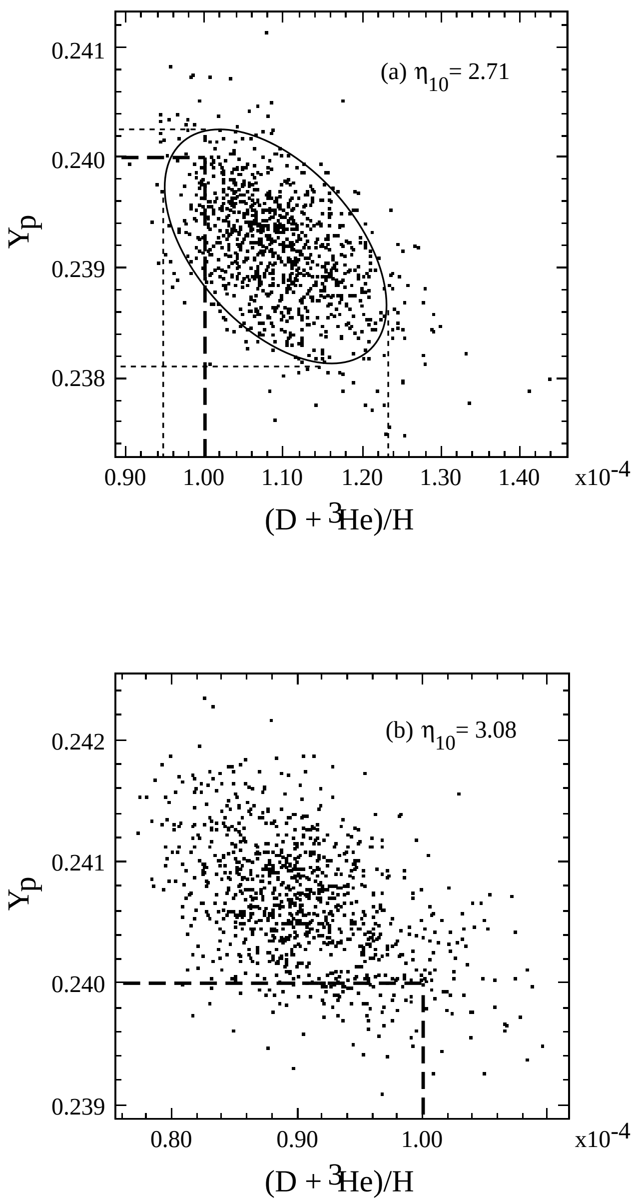

Figure 10. Monte Carlo predictions for the ^{4}He versus the D+^{3}He abundances (taking $N_\nu = 3$ and $\tau_{\rm n} = 889 \pm 2.1$ sec) for (a) $\eta = 2.71 \times 10^{-10}$ and (b) $\eta = 3.08 \times 10^{-10}$. The dashed lines indicate the adopted "reasonable" observational upper bounds. In panel (a), a gaussian contour with $\pm 2\sigma$ limits (dotted lines) on each variable is also shown (from Kernan and Krauss 1994).

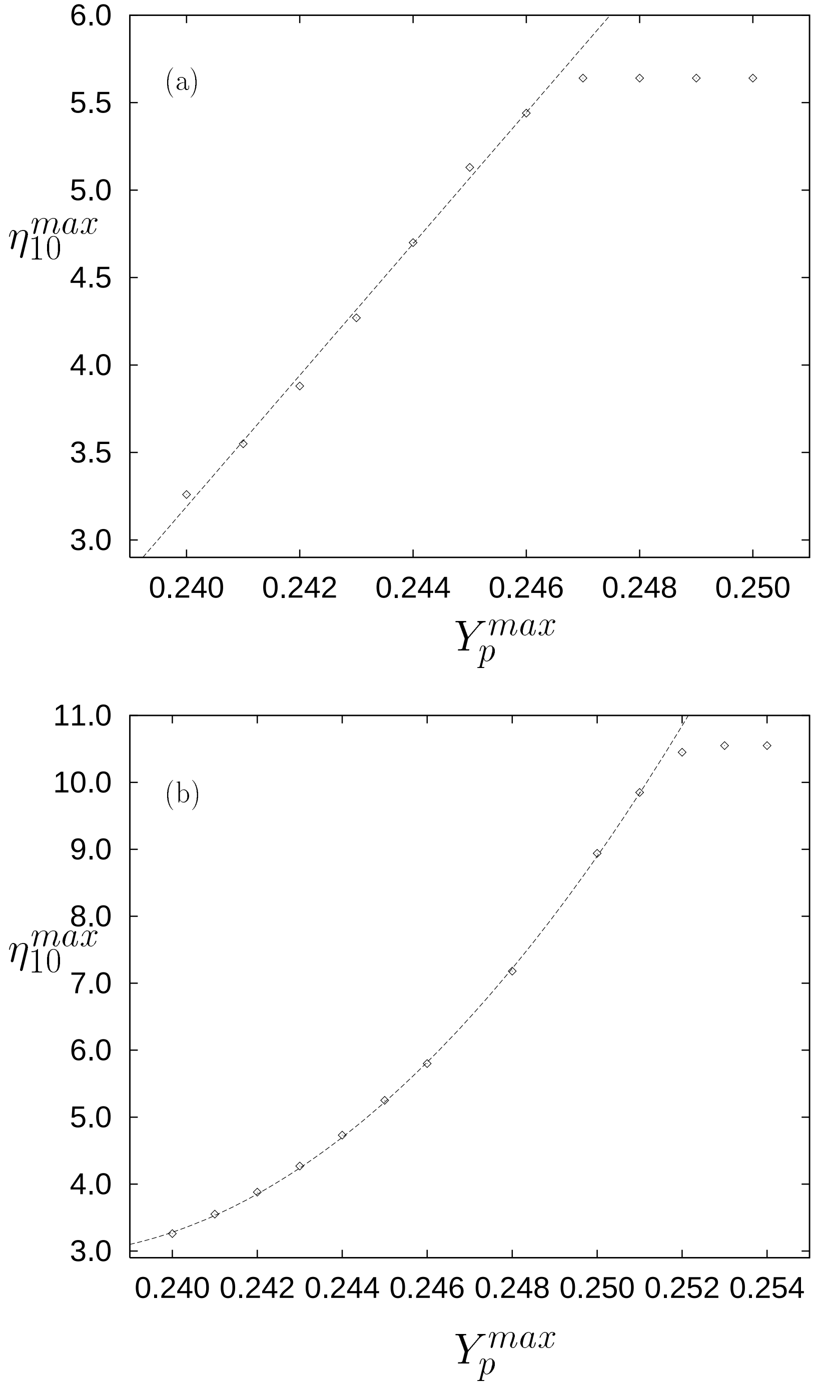

Figure 11. The upper limit to the nucleon-to-photon ratio (in units of 10^{-10}) implied by the ISM bound $\mathrm{D/H} > 1.1 \times 10^{-5}$ combined with (a) the Pop II bound $^7\mathrm{Li/H} < 2.6 \times 10^{-10}$, and (b) the Pop I bound $^7\mathrm{Li/H} < 1.5 \times 10^{-9}$, as a function of the maximum ^{4}He mass fraction.

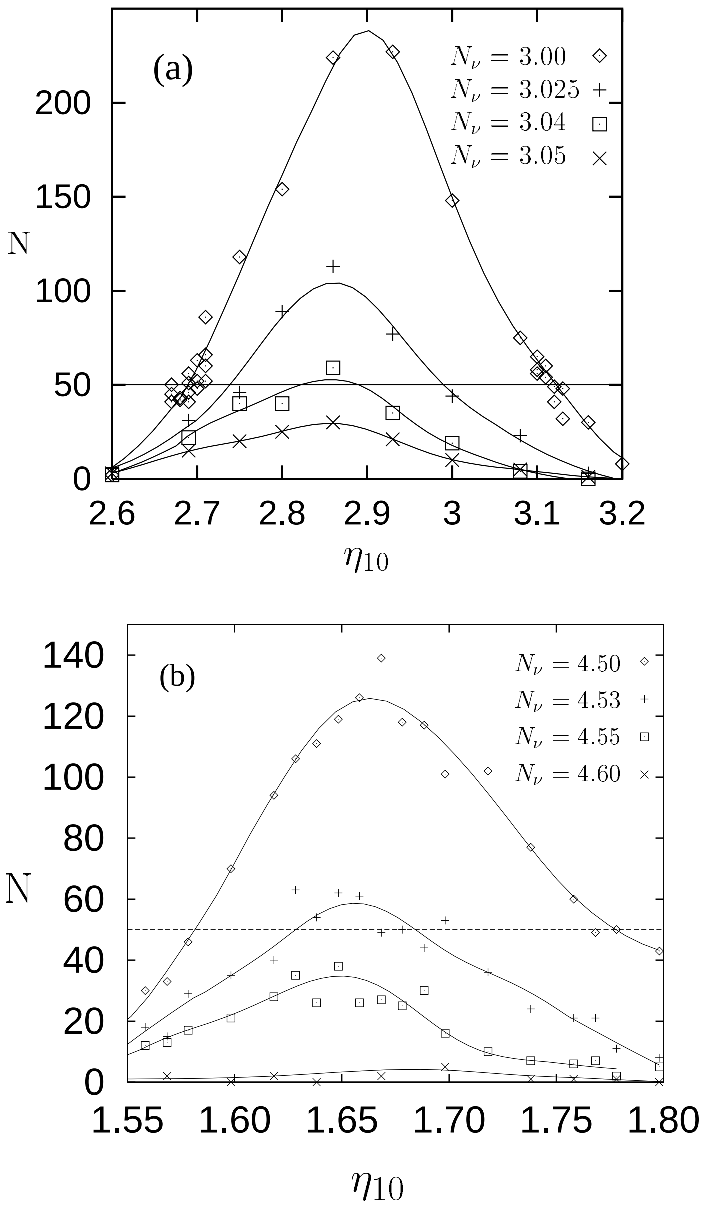

Figure 12. Number of Monte Carlo runs (out of 1000) which simultaneously satisfy the assumed abundance bounds, as a function of η (in units of 10^{-10}), for various values of N_ν. Panel (a) is obtained adopting $Y_{\rm p}(^4{\rm He}) \leq 0.24$ and $[({\rm D} + {}^3{\rm He})/{\rm H}]_{\rm p} \leq 10^{-4}$ (from Kernan and Krauss 1994) while panel (b) is obtained taking $Y_{\rm p}(^4{\rm He}) \leq 0.25$, $[{\rm D/H}]_{\rm p} \leq 2.5 \times 10^{-4}$ and $[^7{\rm Li/H}]_{\rm p} \leq 2.6 \times 10^{-10}$ (from Kernan and Sarkar 1996).

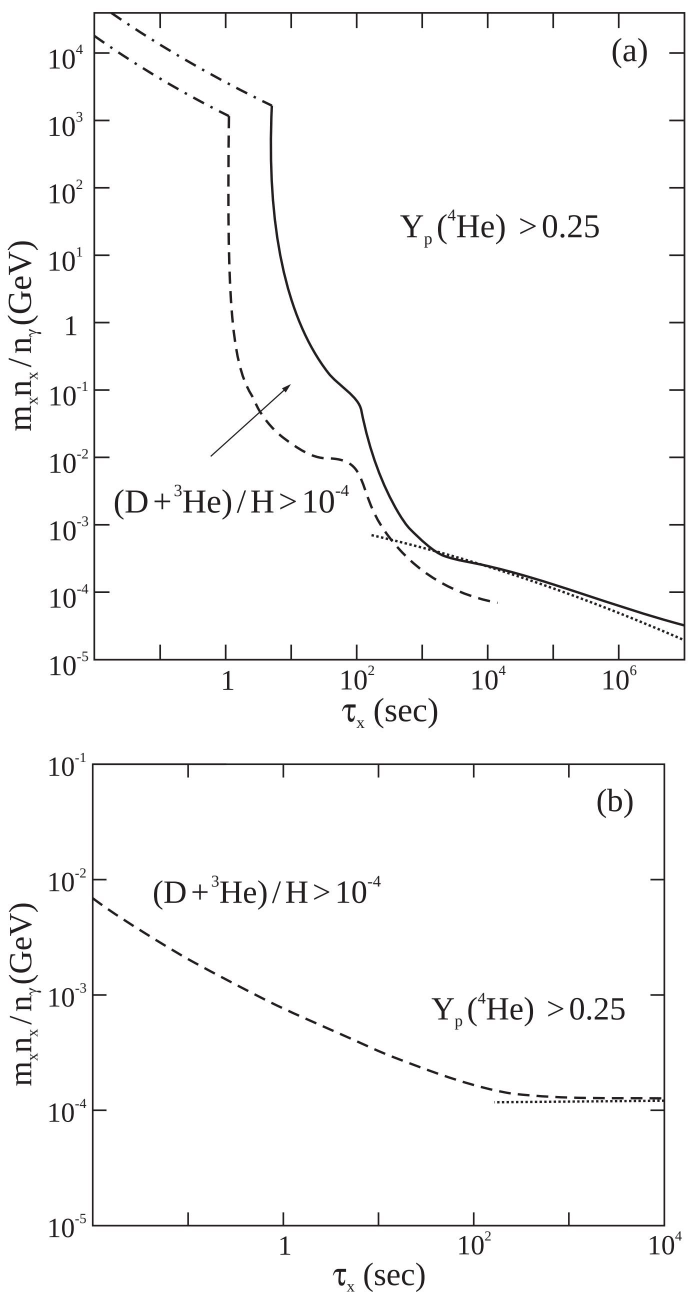

Figure 13. Upper bounds on the decaying-particle abundance as a function of its lifetime obtained from considerations of (a) entropy generation and increase in the expansion rate and (b) increase in the expansion rate alone (for ‘invisible’ decays). Above the full lines ^{4}He is overproduced whereas above the dashed lines $D + {}^3$He is overproduced; the dot-dashed lines in panel (a) assume in addition that the initial nucleon-to-photon ratio is less than 10^{-4} (Scherrer and Turner 1988a,b). The dotted lines indicate the approximate bounds given by Ellis *et al* (1985b).

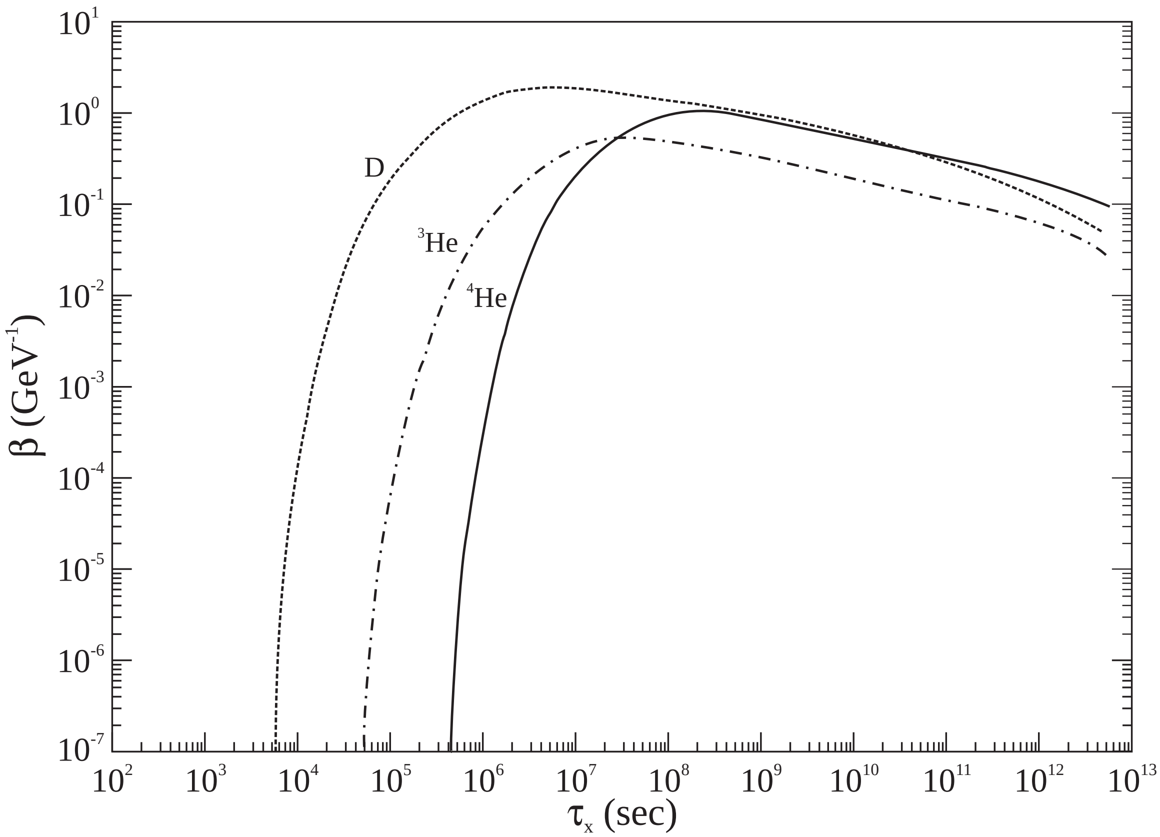

Figure 14. Normalized rates for photodissociation of light elements (as labelled) by electromagnetic cascades generated by massive unstable particles, as a function of the particle lifetime (Ellis *et al* 1992).

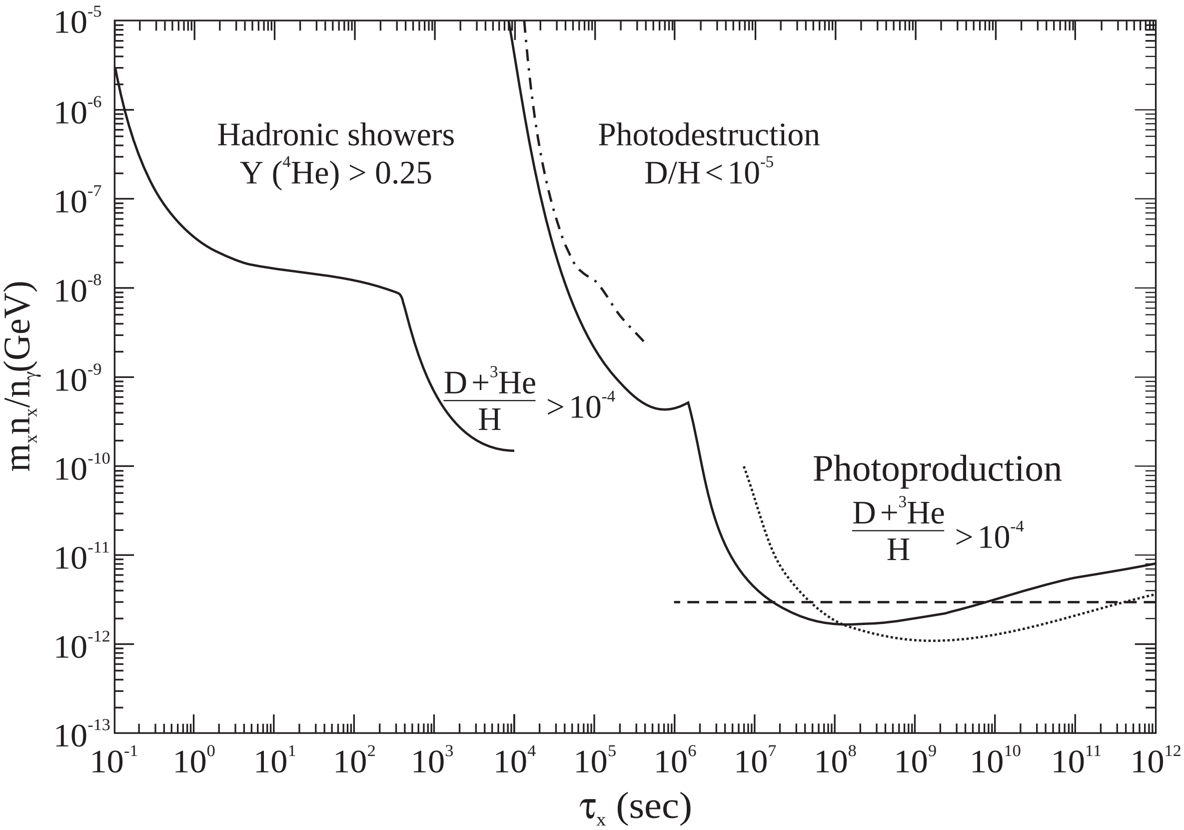

Figure 15. Upper bounds on the abundance of an unstable particle as a function of its lifetime from the effects of electromagnetic (Ellis *et al* 1992) and hadronic cascades (Reno and Seckel 1988) on the primordially synthesized abundances. Other results shown are from Dimopoulos *et al* (1989) (dot-dashed line), Ellis *et al* (1985b) (dashed line) and Protheroe *et al* (1995) (dotted line).

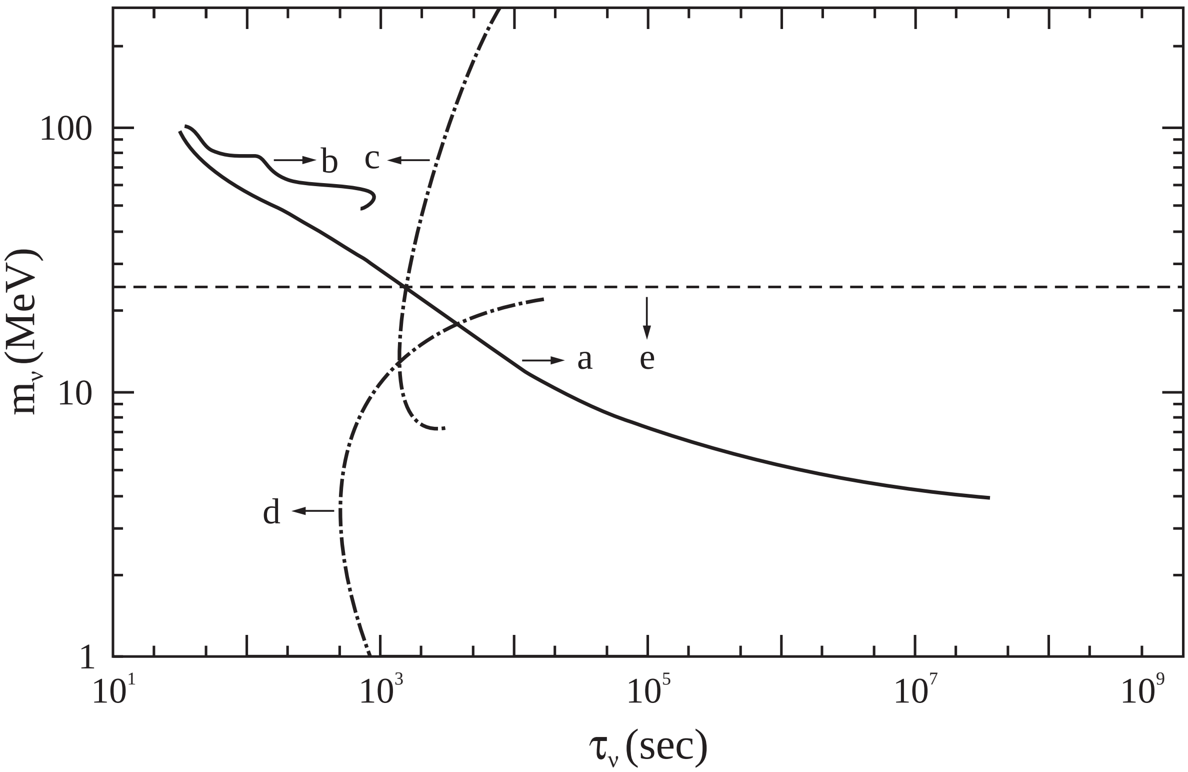

Figure 16. Upper bounds (dot-dashed curves) on the lifetime for $\nu_\tau \to \mathrm{e}^-\mathrm{e}^+\nu_\mathrm{e}$ (or $\nu_\tau \to \nu_{\mathrm{e},\mu}\gamma$) from nucleosynthesis compared with lower bounds (full lines) from laboratory experiments (updated from Sarkar and Cooper 1984). Curves (a) and (b) are calculated from limits on the mixing angle $|U_{\mathrm{e}3}|^2$ obtained from, respectively, searches for additional peaks in $\pi \to \mathrm{e}\nu$ decay (Britton *et al* 1992, De Leener-Rosier 1991) and measurement of the branching ratio $\pi \to \mathrm{e}\nu/\pi \to \mu\nu$ (Britton *et al* 1994). Curve (d) is the bound from entropy production and speed-up of the expansion rate, while curve (e) is obtained from consideration of deuterium photofission. Curve (c) is the present experimental bound on the ν_τ mass (Busculic *et al* 1995). Note that there is *no* allowed region for an unstable tau neutrino with mass exceeding 1 MeV.

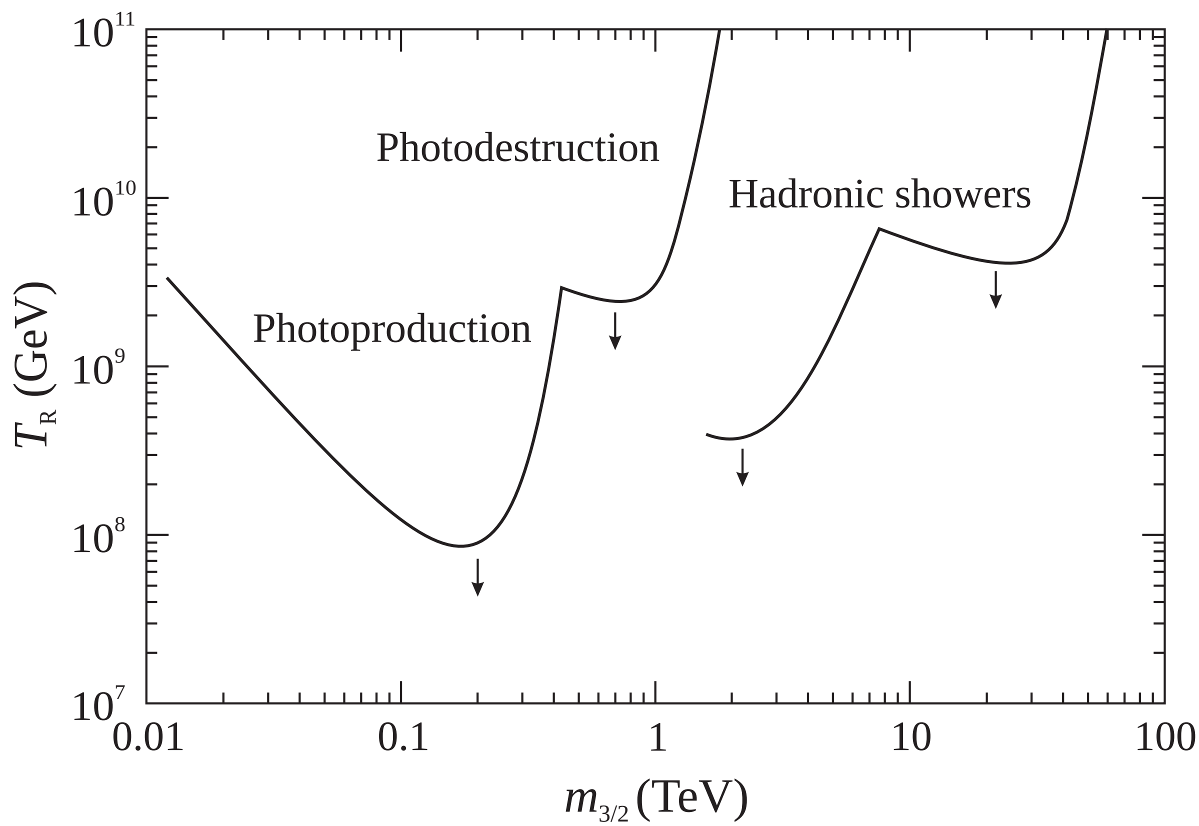

Figure 17. Nucleosynthesis bounds on the reheating temperature after inflation, from consideration of the generation of massive unstable gravitinos (Ellis *et al* 1984b) and the effects of their hadronic (Reno and Seckel 1988) and radiative (Ellis *et al* 1992) decays on elemental abundances.

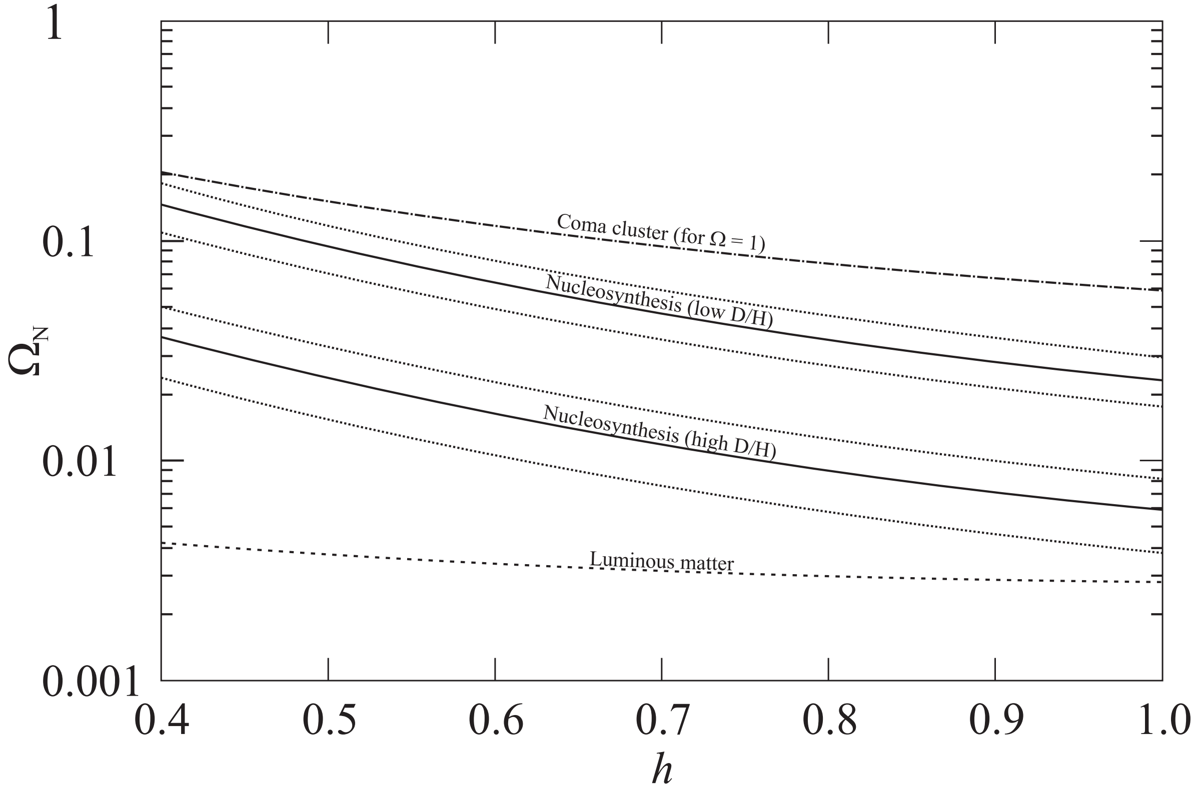

Figure 18. The contribution of nucleons to the cosmological density parameter as a function of the assumed Hubble parameter (after Hogan 1994). The full lines (with dotted '2σ' error bands) show the standard BBN values according as whether the primordial D abundance is taken to be the high value (Songaila *et al* 1994, Rugers and Hogan 1996a,b) or the low value (Tytler *et al* 1996, Burles and Tytler 1996) measured in QAS. The dashed line is the lower limit from an audit of luminous matter in the universe (Persic and Salucci 1992). The dot-dashed line indicates the value deduced from the observed luminous matter in the *Coma* cluster (White *et al* 1993) for $\Omega = 1$; it should be lowered by a factor of Ω^{-1} for $\Omega < 1$.