School of Civil Engineering

Civil Engineering

ETE - Jun 2023

Time : 3 Hours **Marks : 100**

Sem IV - C1UC421T - Probability and Statistics

Your answer should be specific to the question asked

Draw neat labeled diagrams wherever necessary

1. *Let X* and Y be two independent random variables and E(X)=5, E(Y)=8 and V(X)=2, V(Y)=2 respectively. Define a random variable Z = 5X + 2Y. Calculate E(*Z) and V(Z). (where E denotes the expectation and V statnds for variance of the variable).* K1 CO1 (5)

2. Describe the applications of probability Distributions in engineering. K2 CO5 (5)

3. Suppose that the number of cars X that pass through a car wash between 4:00 P.M. and 5:00 P.M. on any sunny Friday has the following probability distribution: K2 CO2 (5)

x	4	5	6	7	8	9
P(X=x)	1/12	1/12	1/4	1/4	1/6	1/6

Let X represent the amount of money, in dollars, paid to the attendant by the manager. Find the attendant's expected earnings for this particular time.

4. Define type I and type II error in hypothesis testing. K3 CO4 (10)
If a certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms, what is the probability that a random sample of 36 of these resistors will have a combined resistance of more than 1458 ohms?

5. Fit a straight line of the form Y=aX +b to the following data: K3 CO2 (10)

X : 0 5 10 15 20 25 30

Y: 10 14 19 25 31 36 39

6. The weekly demand for a drinking-water product, in thousands of liters, from a local chain of efficiency stores is a continuous random variable X having the probability density K2 CO1 (10)

$$f(x) = \begin{cases} 2(x-1) & 1 < x < 2 \\ 0 & else \end{cases}$$

Find the mean and variance of X.

7) K4 CO2 (10)

The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that (a) at least 10 survive, (b) from 3 to 8 survive?

OR

If a random sample $X_i \sim N(\mu, \sigma^2),\ i = 1, 2, ..., n$ then prove that $\bar{X} \sim N(\mu, \sigma^2/n)$, where $\bar{X} = \frac{\sum X_i}{n}$ is sample mean, $N(\mu, \sigma^2)$ denote the normal distribution with mean μ and variance σ^2. K4 CO5 (10)

8. Show that the sample mean $\bar{x}$ of a random sample $x_1, x_2, \ldots, x_n$ is the maximum likelihood estimate for the parameter θ of a Poisson distribution with pmf K5 CO3 (15)

$$P(X = x) = f(x, \theta) = \frac{e^{-\theta}\theta^x}{x!}, \quad x = 0, 1, 2, \ldots$$

9. Ten recruits were subjected to a selection test to ascertain their suitability for a certain course of training. At the end of training they were given a proficiency test. The marks secured by recruits in the selection test (X) and in the proficiency test (Y) are given below: K4 CO5 (15)

X:	10	15	12	17	13	16	24	14	22	20
Y:	30	42	45	46	33	34	40	35	39	38

Calculate Spearman (rank) correlation coefficient for the given data.

PTO

10) The theory predicts the proportion of beans in the four groups A, B, C and D should be 9:3:3:1. In an experiment among 1600 beans, the numbers in the four groups were 882, 313, 287 and 118. Does the experimental result support the theory? [χ20.05 for 3 d.f. = 7·815] K5 CO4 (15)

OR

A machinist is making engine parts with axle diameters of 0·700 inch. A random sample of 10 parts shows a mean diameter of 0·742 inch with a standard deviation of 0·040 inch. Compute the statistic you would use to test whether the work is meeting the specifications. Also find the 95% confidence interval in which most of the mean axle diameter lie. [Given that Tabuated $t_{0.05}$ for (10 - 1) i.e., 9 d.f. for two-tailed test is 2·262]. K5 CO5 (15)